

# Cluster structure prediction in USPEX: basics, examples and recent developments

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1. New variable-composition method for cluster structure prediction
2. Application to  $\text{Si}_n\text{O}_m$  clusters in wide area of compositions:  $1 \leq n \leq 15$ ,  $0 \leq m \leq 20$
3. Charge trapping in  $\text{Cd}_n\text{Se}_m$  clusters

# Fixed composition cluster structure prediction

Theoretical structure prediction of nanoclusters is a very complicated problem:

1. A large number of atoms in a cluster to consider
2. In most cases the real cluster structure differs from the model structures either cut from bulk lattice or built considering some point symmetry. Low symmetry of the system is possible
3. The experimental data on the structure of nanoclusters is also very limited

# Fixed composition cluster structure prediction

In the current version the **000** method is available

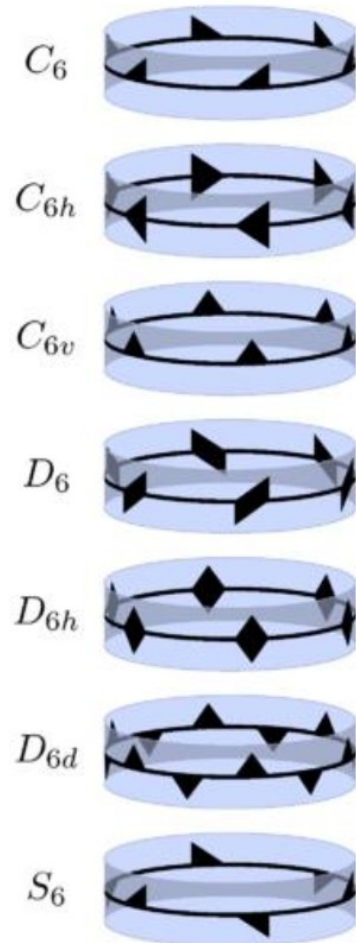
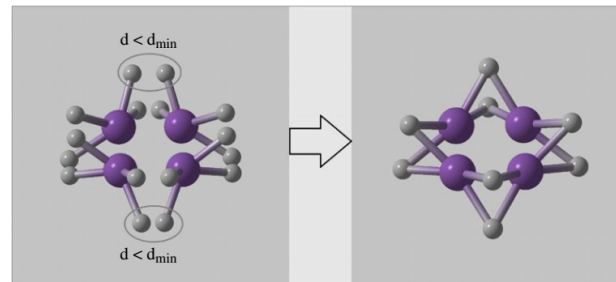
1. 1-st generation is initialized randomly, with clusters obeying one of 32 crystal point group plus some important non-crystallic point groups:

*Crystallographic point groups:*

Hermann-Mauguin	Schönflies	In USPEX
1	C <sub>1</sub>	C1 or E
2	C <sub>2</sub>	C2
222	D <sub>2</sub>	D2
4	C <sub>4</sub>	C4
3	C <sub>3</sub>	C3
6	C <sub>6</sub>	C6
23	T	T
$\bar{1}$	S <sub>2</sub>	S2
M	C <sub>1h</sub>	Ch1
mm2	C <sub>2v</sub>	Cv2
2	S <sub>4</sub>	S4
$\bar{3}$	S <sub>6</sub>	S6
$\bar{6}$	C <sub>3h</sub>	Ch3
m $\bar{3}$	T <sub>h</sub>	Th
2/m	C <sub>2h</sub>	Ch2
mmm	D <sub>2h</sub>	Dh2
4/m	C <sub>4h</sub>	Ch4
32	D <sub>3</sub>	D3
6/m	C <sub>6h</sub>	Ch6
432	O	O
422	D <sub>4</sub>	D4
3m	C <sub>3v</sub>	Cv3
622	D <sub>6</sub>	D6
$\bar{4}3m$	T <sub>d</sub>	Td
4mm	C <sub>4v</sub>	Cv4
$\bar{3}m$	D <sub>3d</sub>	Dd3
6mm	C <sub>6v</sub>	Cv6
m $\bar{3}m$	O <sub>h</sub>	Oh
$\bar{4}2m$	D <sub>2d</sub>	Dd2
$\bar{6}2m$	D <sub>3h</sub>	Dh3
4/mmm	D <sub>4h</sub>	Dh4
6/mmm	D <sub>6h</sub>	Dh6
m $\bar{3}m$	O <sub>h</sub>	Oh

*Important non-crystallographic point groups*

Hermann-Mauguin	Schönflies	In USPEX
5	C <sub>5</sub>	C5
5/m	S <sub>5</sub>	S5
$\bar{5}$	S <sub>10</sub>	S10
5m	Cv <sub>5v</sub>	Cv5
$\bar{10}$	Ch <sub>5h</sub>	Ch5
52	D <sub>5</sub>	D5
$\bar{5}m$	D <sub>5d</sub>	Dd5
$\bar{10}2m$	D <sub>5h</sub>	Dh5
532	I	I
$\bar{5}3m$	I <sub>h</sub>	Ih



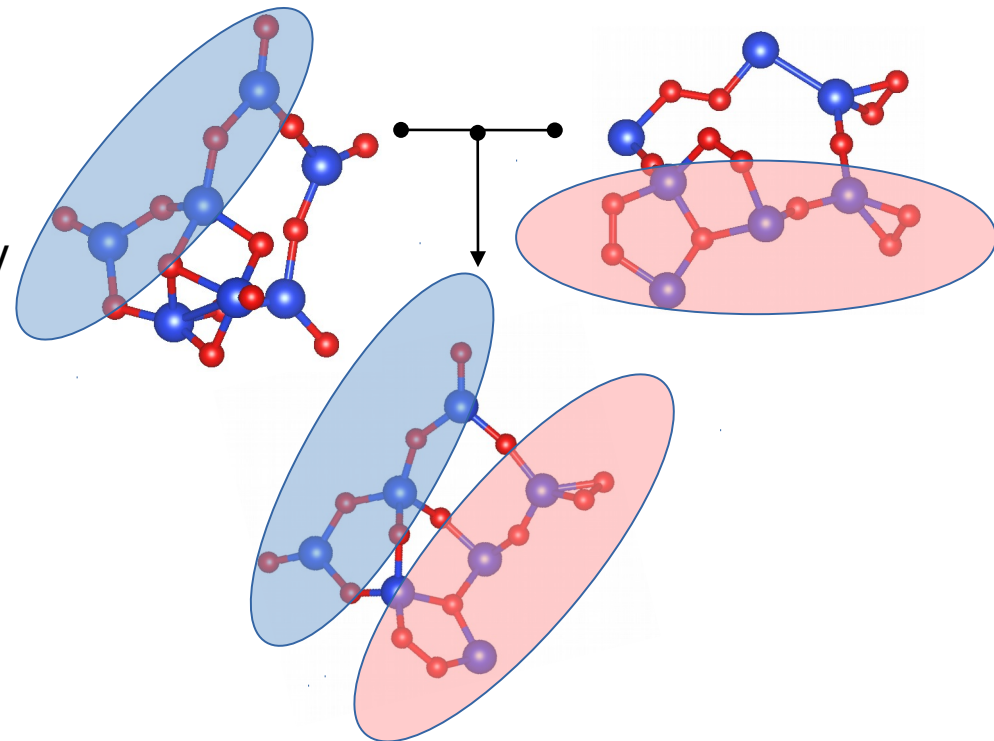
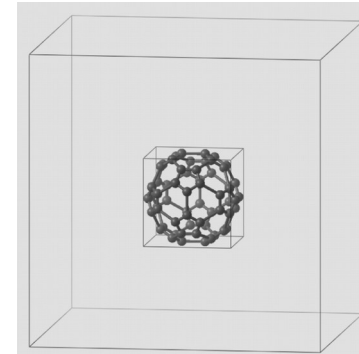
# Fixed composition cluster structure prediction

2. The following generations are produced via similar operators to 3D, but with some important differences

1. Usually supercell approach is applied
2. In that case all variation operators are applied to the “small cell”
3. The fingerprint vector is changed:

$$F_{A_i B}(R) = \sum_{B_j} \frac{\delta(R - R_{ij})}{4\pi R_{ij}^2 N_B \Delta}$$

4. Slab-by-slab heredity is replaced by the cut-and-splice operator



# For practical applications tens or hundreds of cluster compositions should be considered:

- in experiments clusters are often formed as ensembles of various sizes and compositions
- to study nucleation processes;
- to find compounds with required properties;
- etc.

The computation of  $N$  different cluster compositions is  $N$  times longer than one individual fixed-composition calculation.

# New method: New search space, new fitness function, new variation operators

1. New search space – the union of search spaces of clusters with different compositions from a user-defined range:

```
% atomType  
Cu Au  
% EndAtomType  
% numSpecies  
6 10  
6 10  
% EndNumSpecies
```

# New method: New search space, new variation operator, new fitness function

## **New fitness function:**

Total energy doesn't work

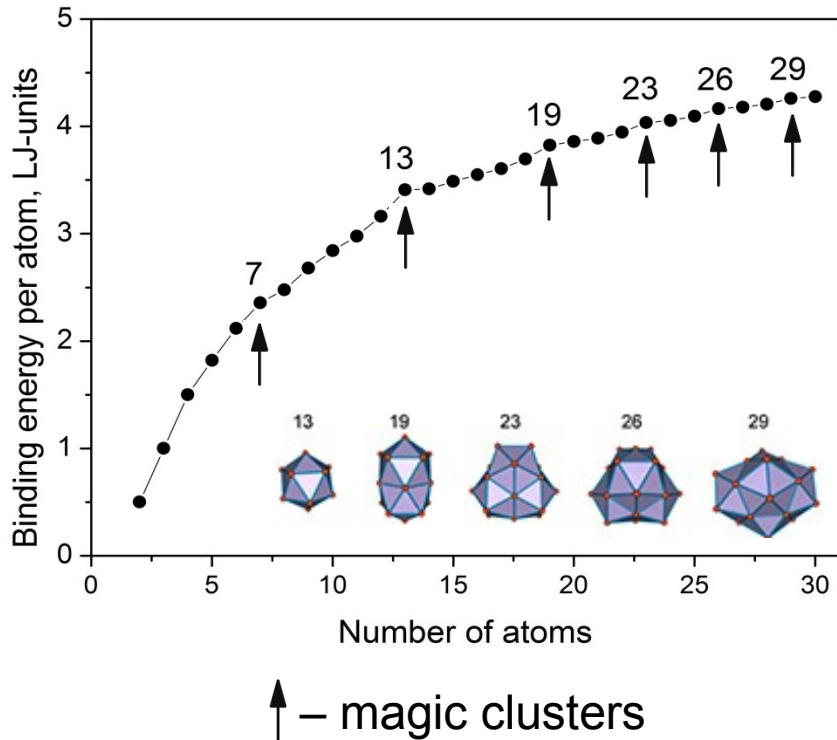
The only thermodynamically stable cluster is an infinite one, i.e. bulk  $\rightarrow$  cohesion, reaction, formation or binding energy don't work either

The solution is to exploit the idea of the **magic clusters** – the most abundant in nature and in the mass spectra of particle beams

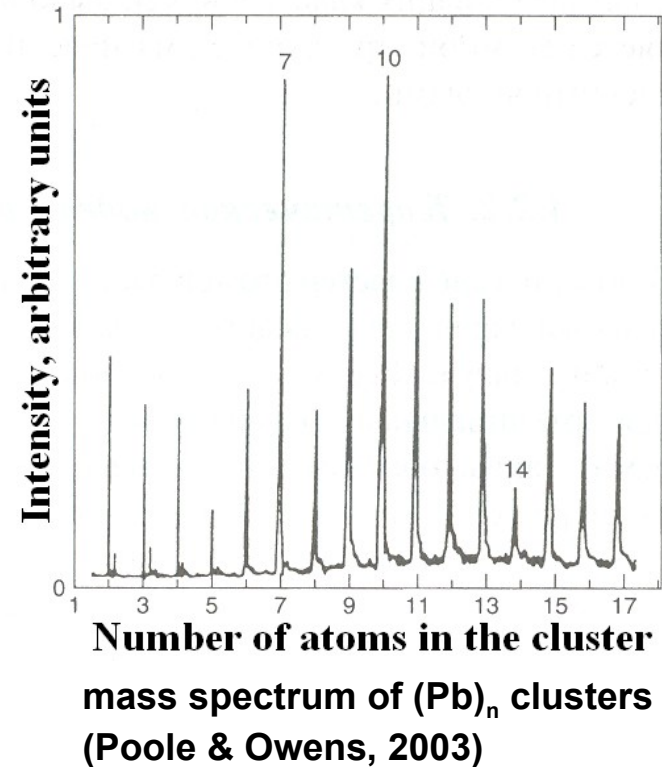


# Stability of the clusters: magic clusters

Model system: Lennard-Jones clusters



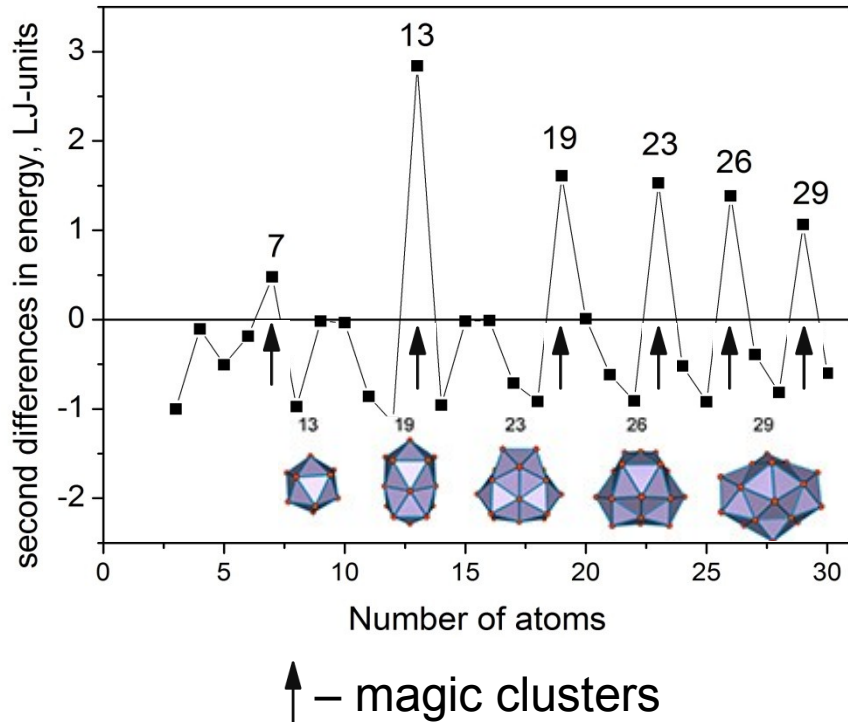
Real system: Pb clusters



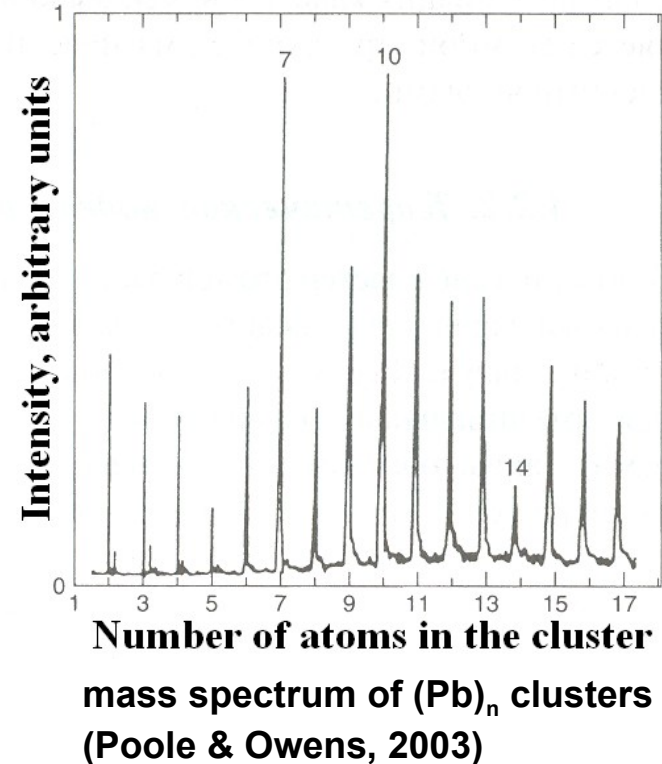
For clusters, stability can be determined with respect to neighboring clusters

# Magic clusters

Model system: Lennard-Jones clusters



Real system: Pb clusters

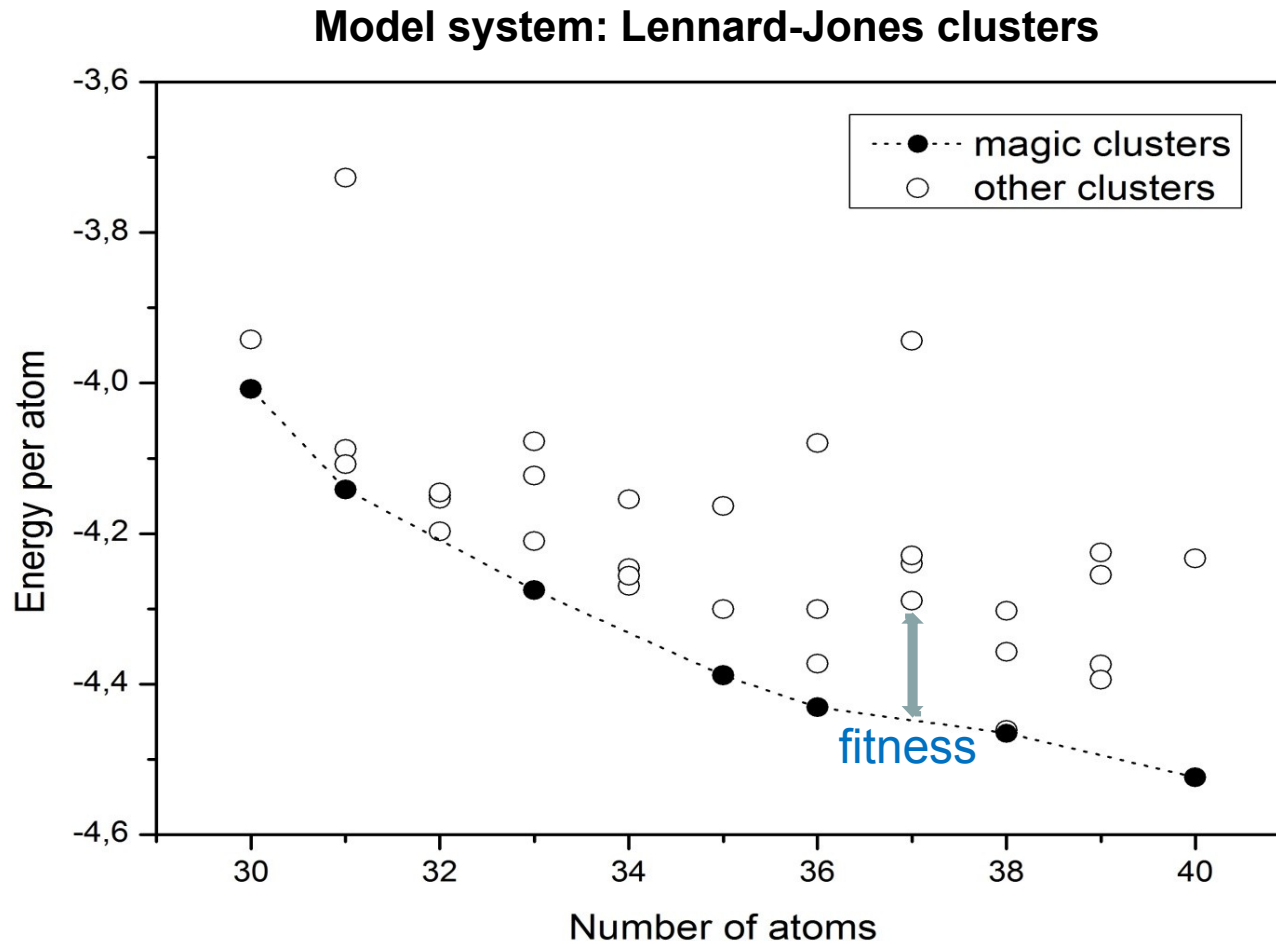


**Stability criterion (magic clusters):**

$$\Delta^2 E = E(n+1) + E(n-1) - 2E(n) > 0, \text{ or}$$
$$E(n) < (E(n+1) + E(n-1))/2$$

# Method: computational scheme; choosing the target function of optimization

New fitness – energy is counted with respect to a line connecting neighboring magic clusters



# Method: variation operators

## Operators in the standard fixed-composition mode:

- Random initialization
- Heredity (parents and offspring of the same composition)
- Softmutation
- Permutation

## New operators in the variable-composition mode:

- Transmutation
- New heredity (parents may have different compositions)
- Add or remove atom

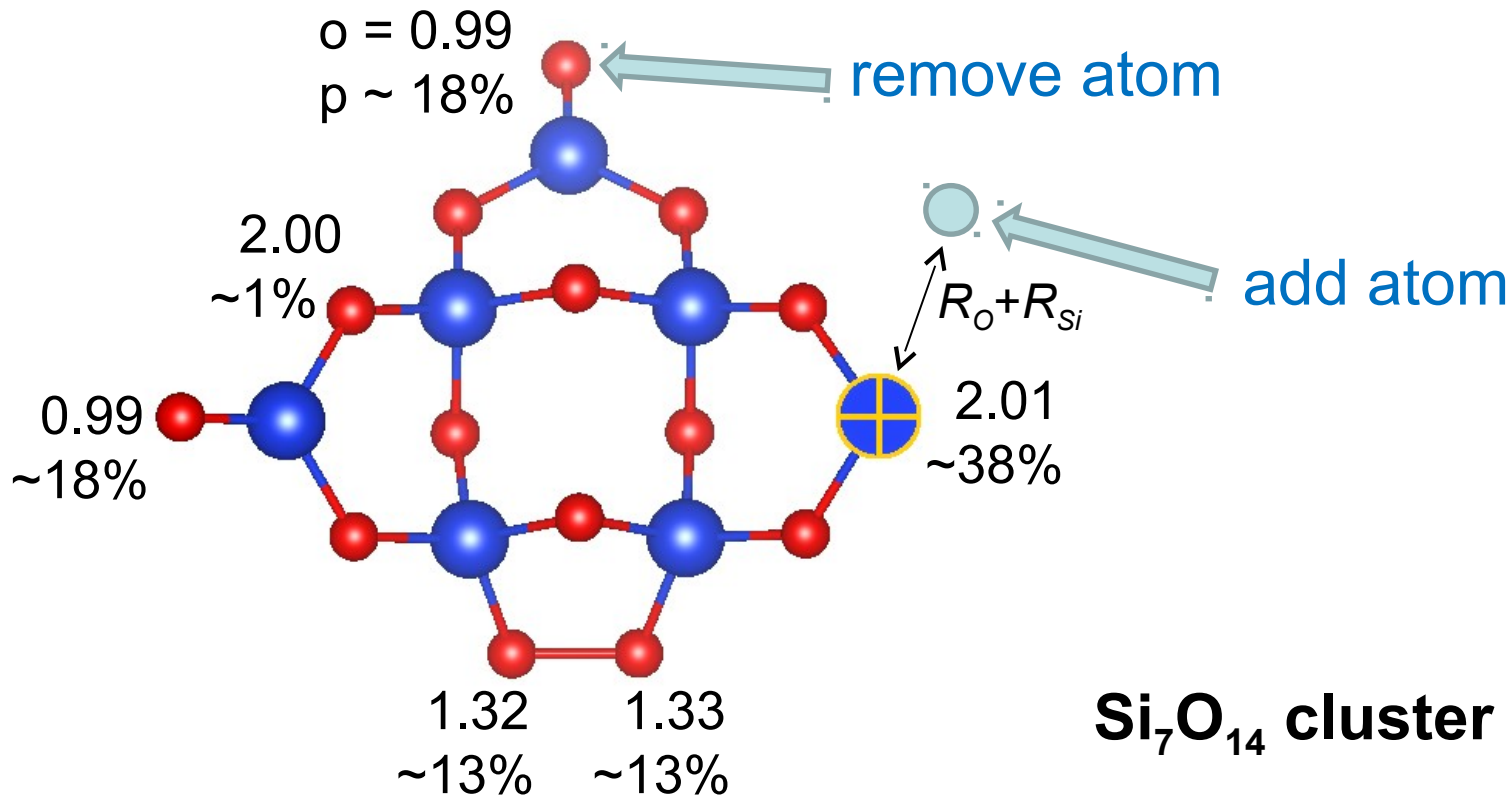
# Add and remove atom

Coordination numbers:  $O_i = \frac{\sum_j (\exp(-(\Delta r_{ij} - R_i - R_j)/0.37))}{\max(\exp(-(\Delta r_{ij} - R_i - R_j)/0.37))}$

$R_i$  - covalent radius of atom  $i$ :

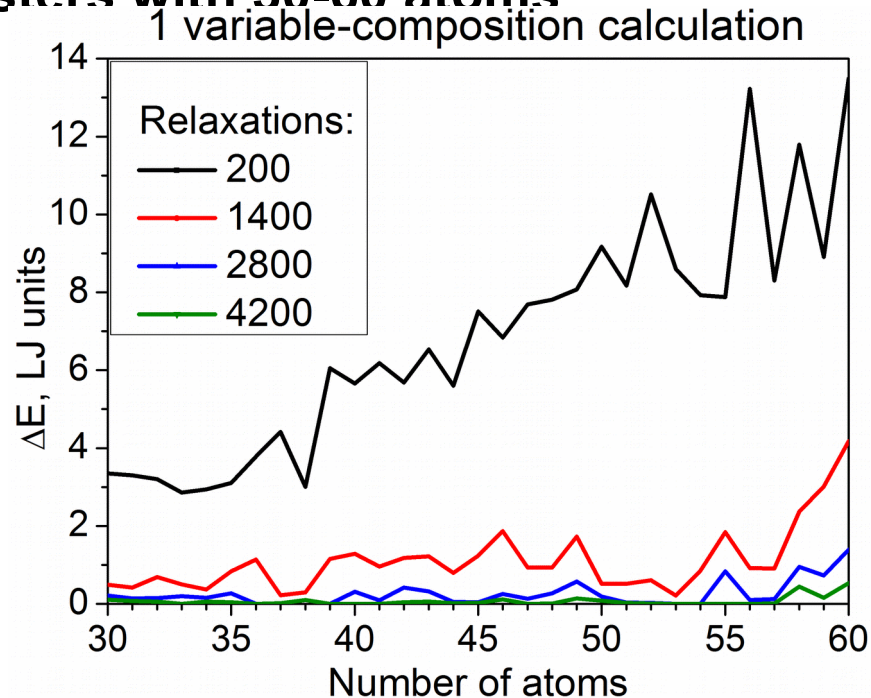
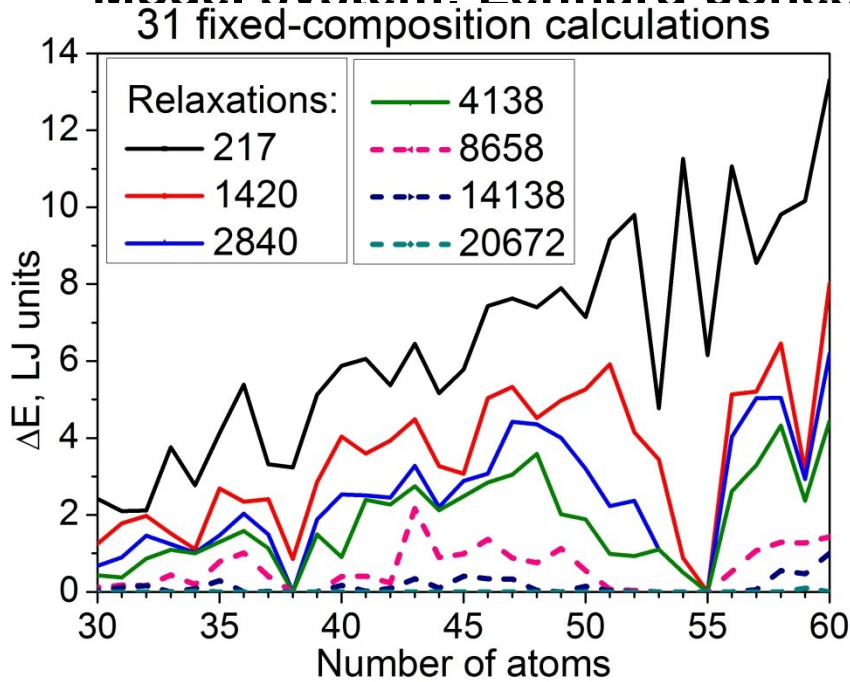
Probability of addition or removal atom (for every type of atom):

$$p_i \sim O_{i_{max}} - O_i$$



# Variable-composition vs. fixed-composition

## Model system: Lennard-Jones clusters with 30-60 atoms



Total number of relaxations:

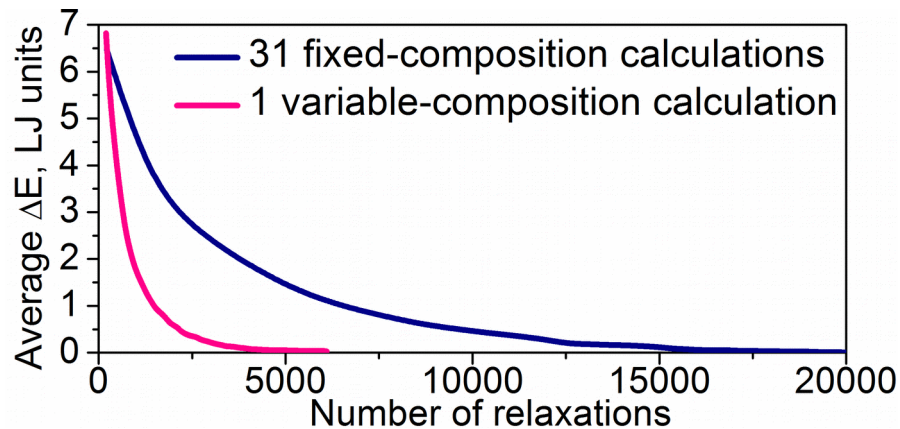
Fixed-composition method: ~20000

Variable-composition method: ~4000

**Speed up is ~5 times**

For binary model clusters:

**Speed up is up to 50 times**



# Ab initio study of Si-O clusters

## Si-O clusters:

- practical importance in many fields:

- astrophysics: formation of silicates from SiO molecules in a circumstellar space;
- experiments on growth of oxide-coated Si nanowires from gas-phase SiO
- Super-oxidized clusters ( $\text{Si}_n\text{O}_m$ ,  $m > 2n$ ) can manifest biological activity.

- very complicated for structure prediction:

crystalline silica alone has 14 structural forms.

# Ground-state structures of Si-O clusters

Si<sub>n</sub>O<sub>m</sub> clusters, 1 ≤ n ≤ 15, 0 ≤ m ≤ 20 (315 different compositions)

USPEX + VASP

20 best structures for each composition are refined with

GAUSSIAN code, B3LYP/6-311+G(2d,p) approach

		<i>m</i>																						
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
<i>n</i>	1						*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	
	2							*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	
	3										*	*	*	*	*	*	*	*	*	*	*	*	*	
	4											*	*	*	*	*	*	*	*	*	*	*	*	
	5													+	*	*	*	*	*	*	*	*	*	
	6															*	*	*	*	*	*	*	*	
	7																							
	8		*	*	*	*	*	*	*	+						*	*	*		*	*	*	*	
	9		*	*	*	*	*	*	*	*	+					*	*	*	*	*		*	*	
	10		*	*	*	*	*	*	*	*	*	+				+	+	*	*	*	*	*	*	
	11		*	*	*	*	*	*	*	*	*	*	+			*	*	*	*	*	*	*	*	
	12		*	*	*	*	*	*	*	*	*	*	*				+	*	*	*	*		*	*
	13		*	*	*	*	*	*	*	*	*	*	*	*		+	+	+	*	*	*	*	*	*
	14		*	*	*	*	*	*	*	*	*	*	*	*	*	*	+	+	+	*	*	*	*	*
	15		*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	+		+	*	*	*	*

## Results:

99 (of 315) clusters – reproduced results of other studies (empty cells);

17 (of 315) clusters – optimal structures were improved (+);

199 (of 315) clusters – were predicted for the first time (\*).



# Stability of binary clusters

## Binary A<sub>n</sub>B<sub>m</sub> clusters

$$\left. \begin{aligned} \Delta_{nn}(n,m) &= E(n+1,m) + E(n-1,m) - 2E(n,m), \\ \Delta_{mm}(n,m) &= E(n,m+1) + E(n,m-1) - 2E(n,m) \end{aligned} \right\}$$

$$\underline{\Delta_{\min}(n,m) = \min\{\Delta_{nn}(n,m), \Delta_{mm}(n,m)\}}$$

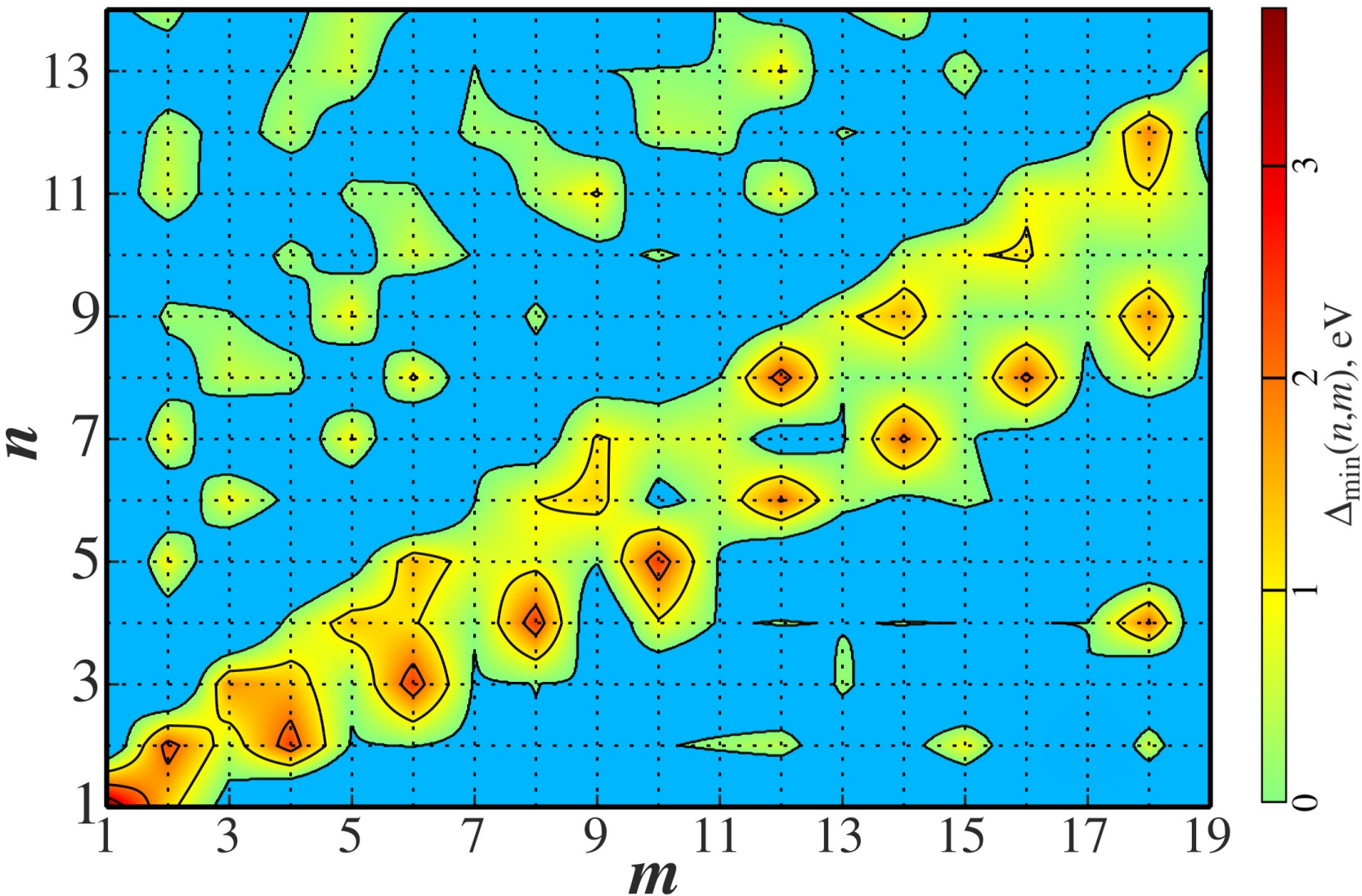
$\Delta_{\min}(n,m) < 0$  – cluster is unstable

$\Delta_{\min}(n,m) > 0$  – cluster is stable (magic)

$\Delta_{\min}(n,m)$  – numerical degree of stability

# Stability map of $\text{Si}_n\text{O}_m$ clusters: $\Delta_{\min}(n,m)$

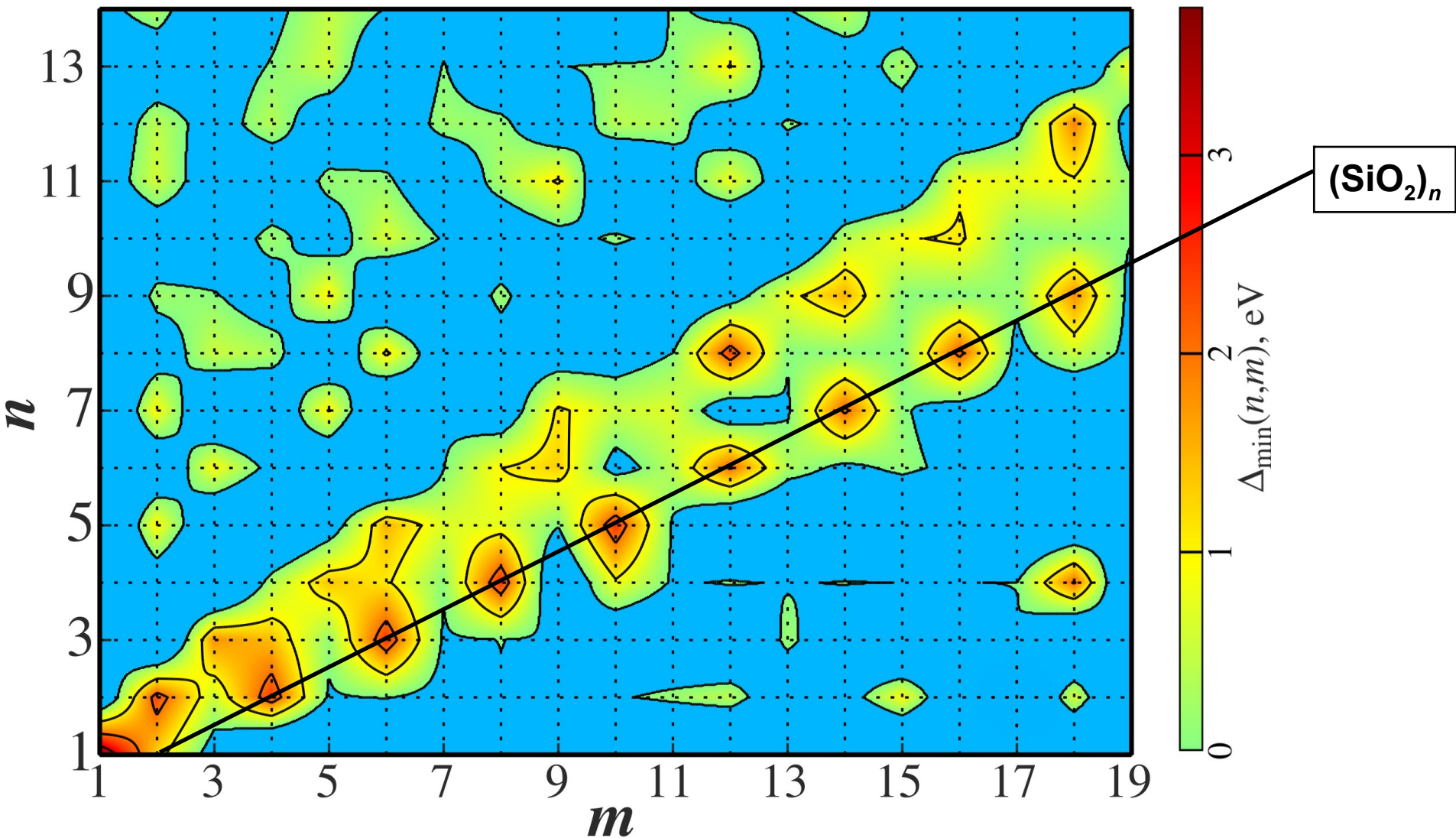
Regions of instability are marked by blue color.



$$\left. \begin{aligned} \Delta_{nn}(n,m) &= E(n+1,m) + E(n-1,m) - 2E(n,m), \\ \Delta_{mm}(n,m) &= E(n,m+1) + E(n,m-1) - 2E(n,m) \end{aligned} \right\} \Delta_{\min}(n,m) = \min\{\Delta_{nn}(n,m), \Delta_{mm}(n,m)\}$$

# Stability map of $\text{Si}_n\text{O}_m$ clusters: $\Delta_{\min}(n,m)$

Regions of instability are marked by blue color.

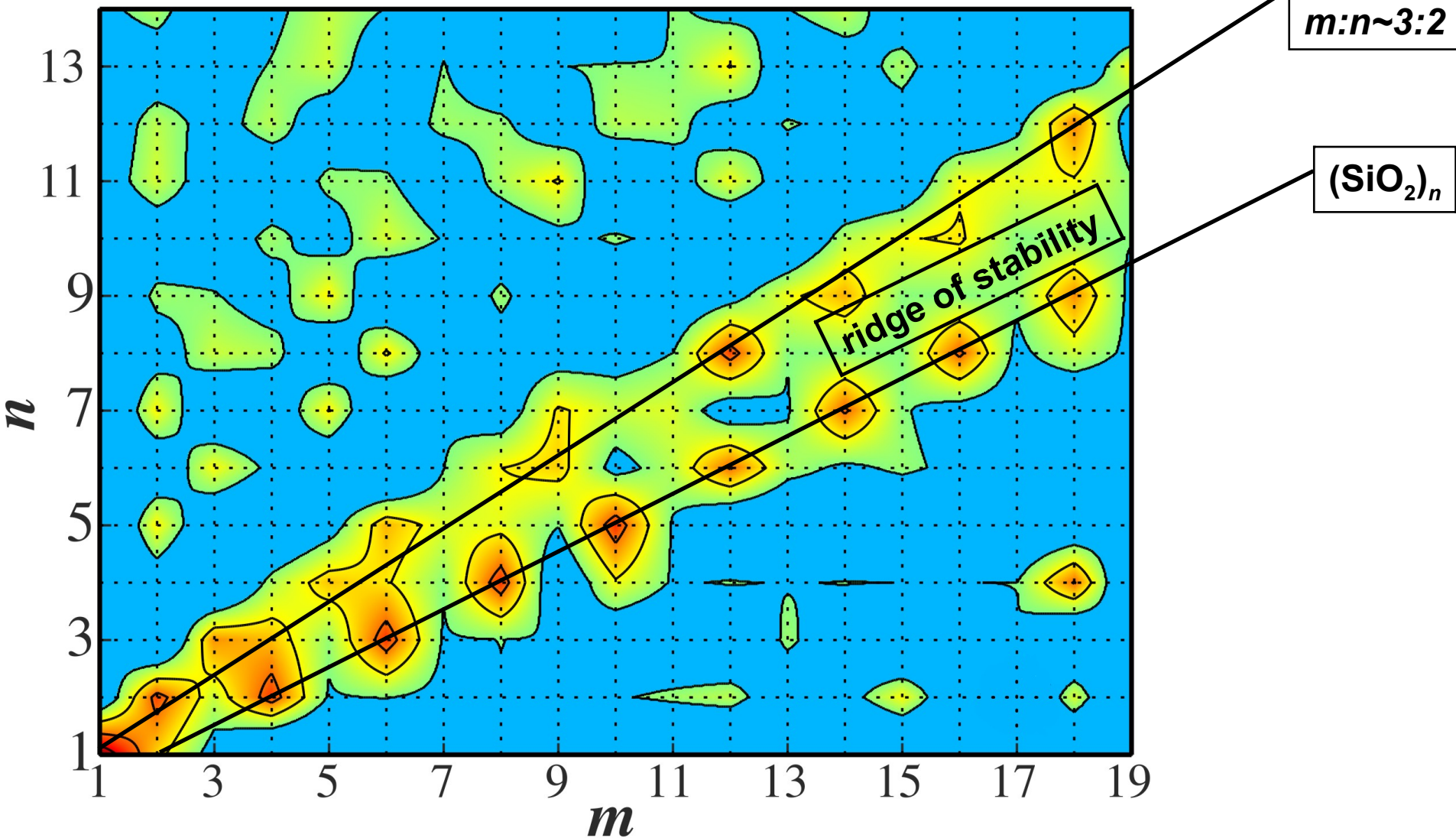


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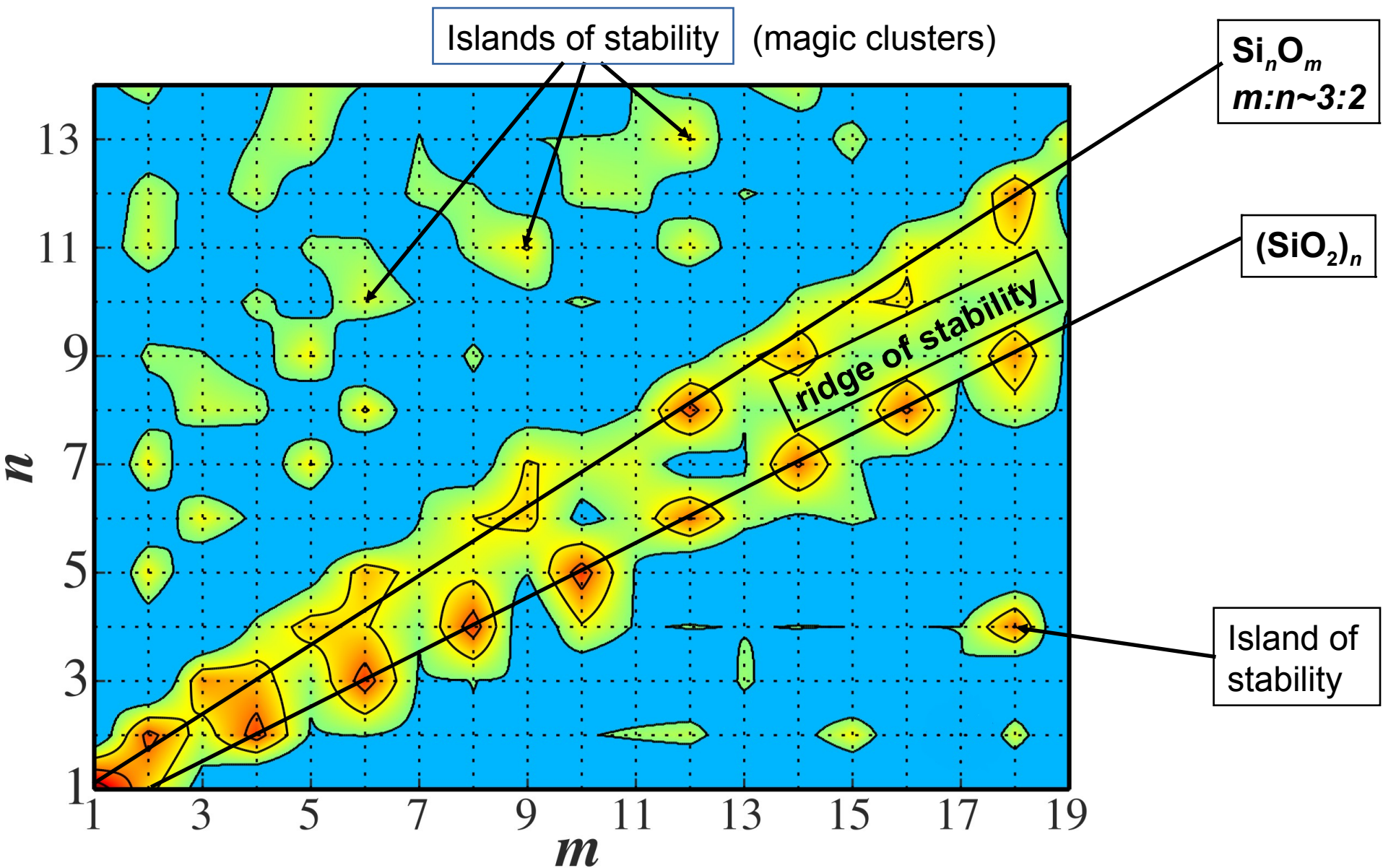
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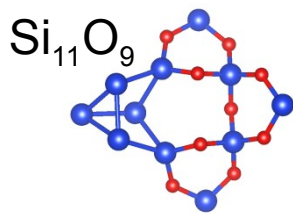
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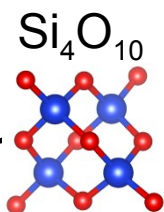
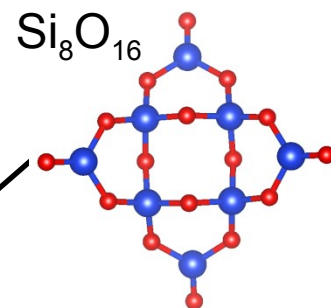
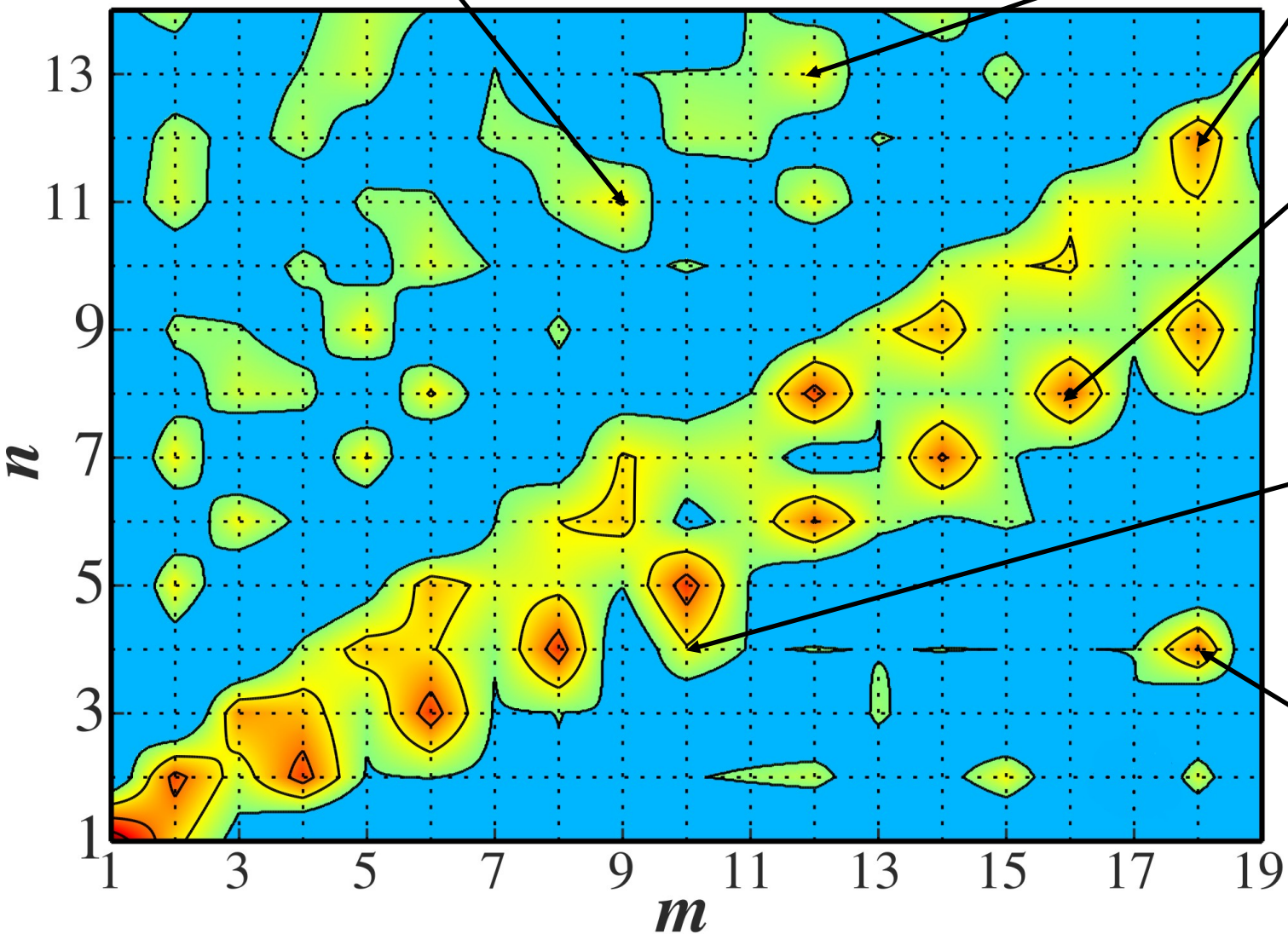
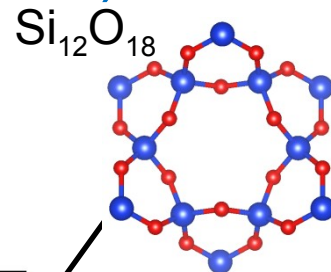
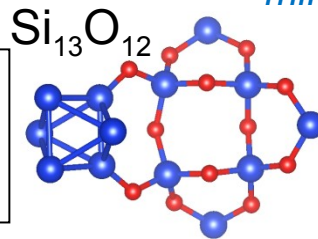
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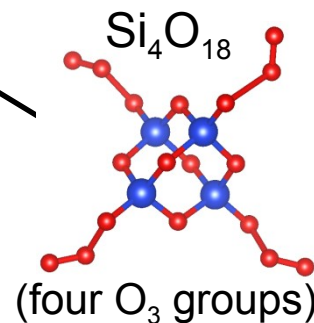
# Stability map of $\text{Si}_n\text{O}_m$ clusters: $\Delta_{\min}(n,m)$



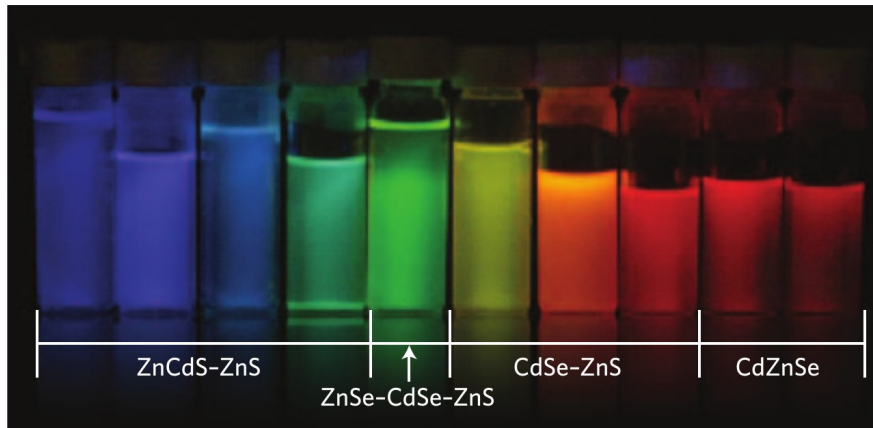
Segregated clusters:  
group of Si atoms and  
skeleton of Si-O bonds



(oxygen radicals)

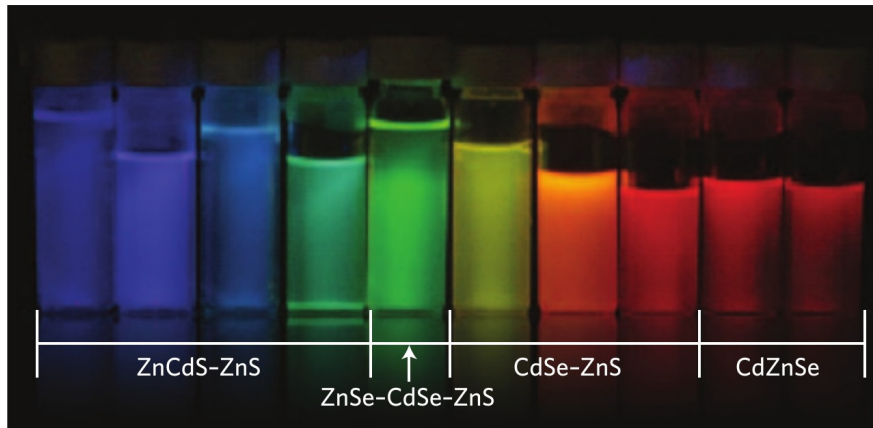


# Charge traps in CdSe clusters



$\lambda$  depends on cluster size making them perfect candidates for LEDs/dyes, however quantum yield is limited by charge trapping (due to localized surface states)

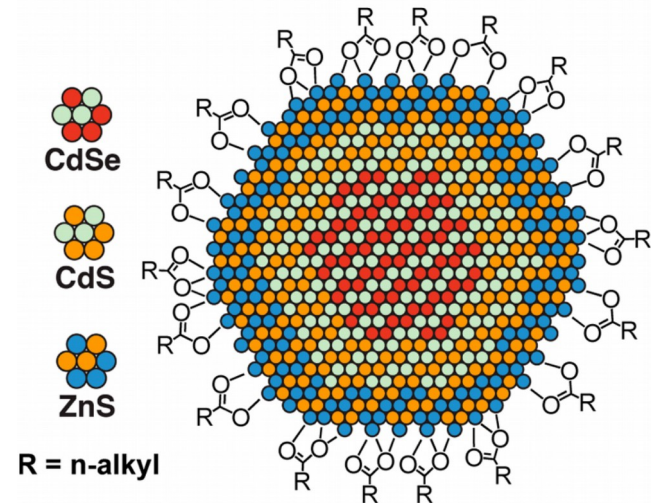
# Charge traps in CdSe clusters



$\lambda$  depends on cluster size making them perfect candidates for LEDs/dyes, however quantum yield is limited by charge trapping (due to localized surface states)

Solutions:

1. Core-shell structures with dielectric shell
2. Passivation by ligands





# Charge traps in $\text{Cd}_n\text{Se}_m$ clusters: final picture

To study charge trapping one must inspect the localization of the near-gap electronic states

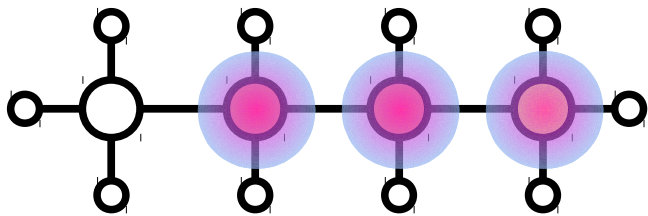
$$\text{PR}_i = \frac{1}{\sum_a P_{i,a}^2}$$

$P_{i,a}$  – contribution of atom  $a$  to  $i$ -th MO,  $\sum_a P_{i,a} = 1$

These are usually taken via Mulliken population analysis:

$\Psi_i = \sum_{\mu} c_{\mu i} \phi_{\mu}$   $i$ -th MO is expanded over basis set of atomic orbitals  $\phi_{\mu}$

$$P_{i,a} = \sum_{\mu \in a} c_{\mu i}^* c_{\nu i} \overbrace{\langle \phi_{\mu} | \phi_{\nu} \rangle}^{\text{overlap integral}}$$

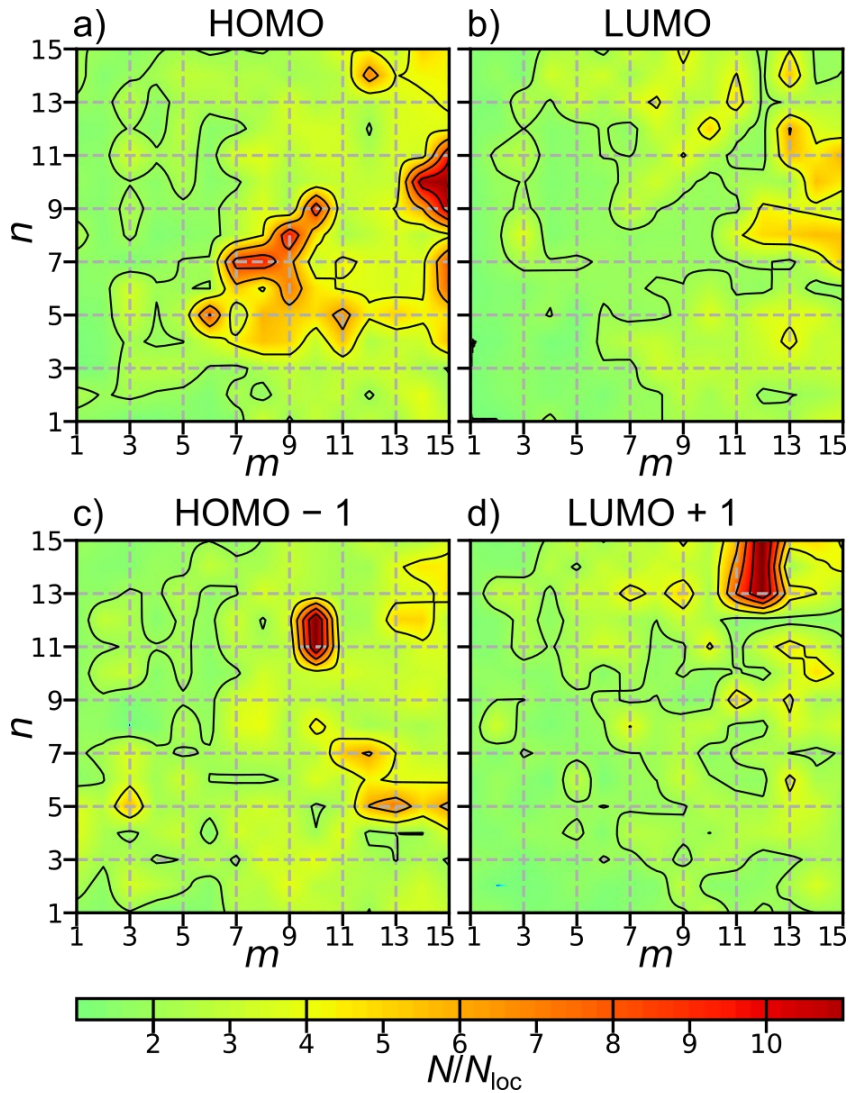


$$P_{i,a} = 0 \quad = 1/3 \quad = 1/3 \quad = 1/3$$

$$\text{PR} = \frac{1}{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = 3 \approx N_{\text{loc}}$$

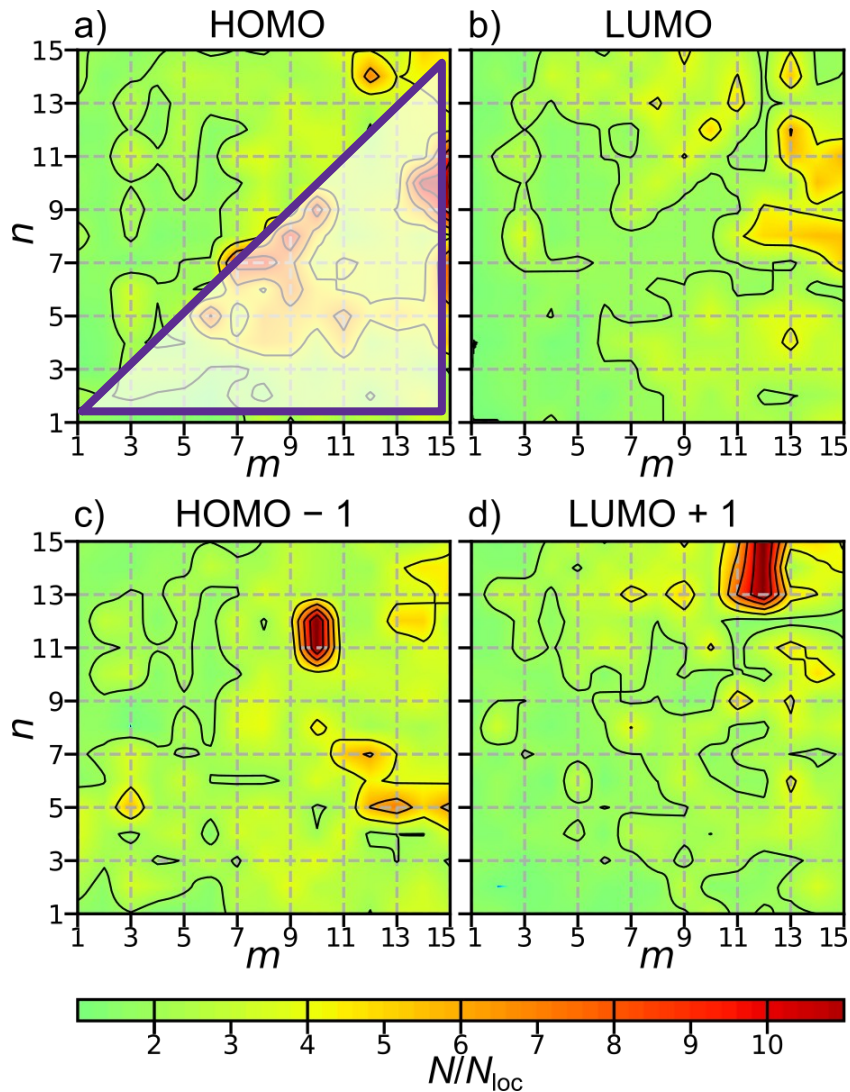
# Maps of the localizations in $\text{Cd}_n\text{Se}_m$ clusters:

The higher  $N/N_{\text{loc}}$  – the more localized state

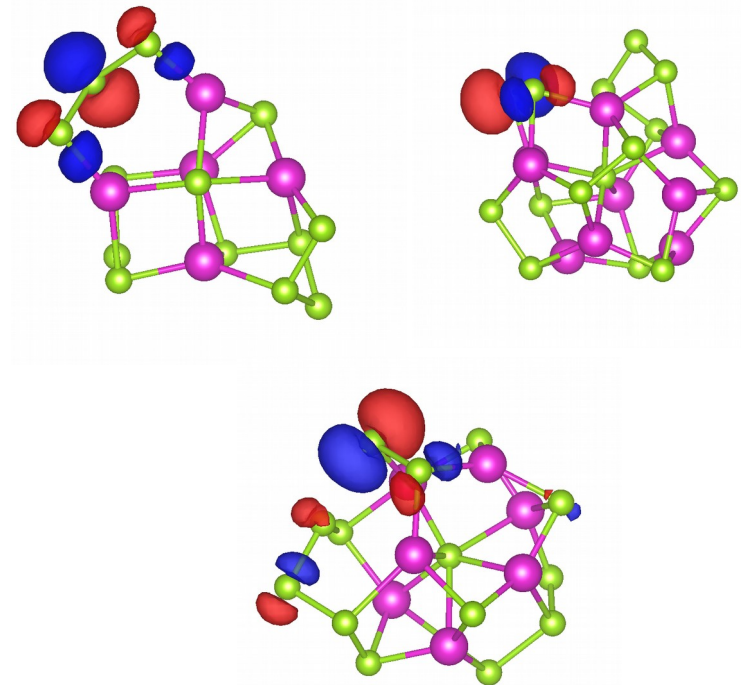


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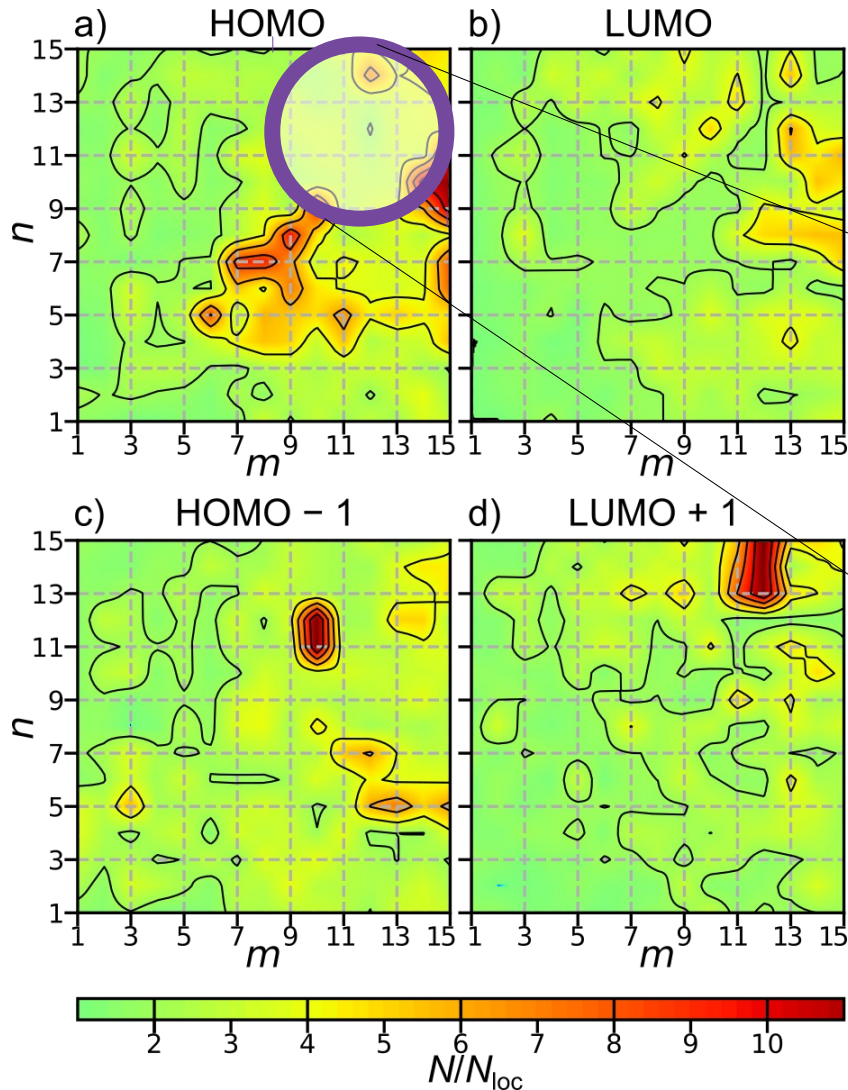


1. Localized states are mostly in lower right triangle – traps are mostly observed in Se-rich clusters



# Maps of the localizations in $\text{Cd}_n\text{Se}_m$ clusters:

The higher  $N/N_{\text{loc}}$  – the more localized state

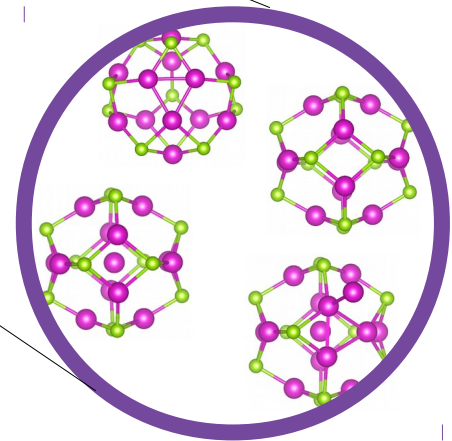


2. Maps are rather **smooth**:

close compositions

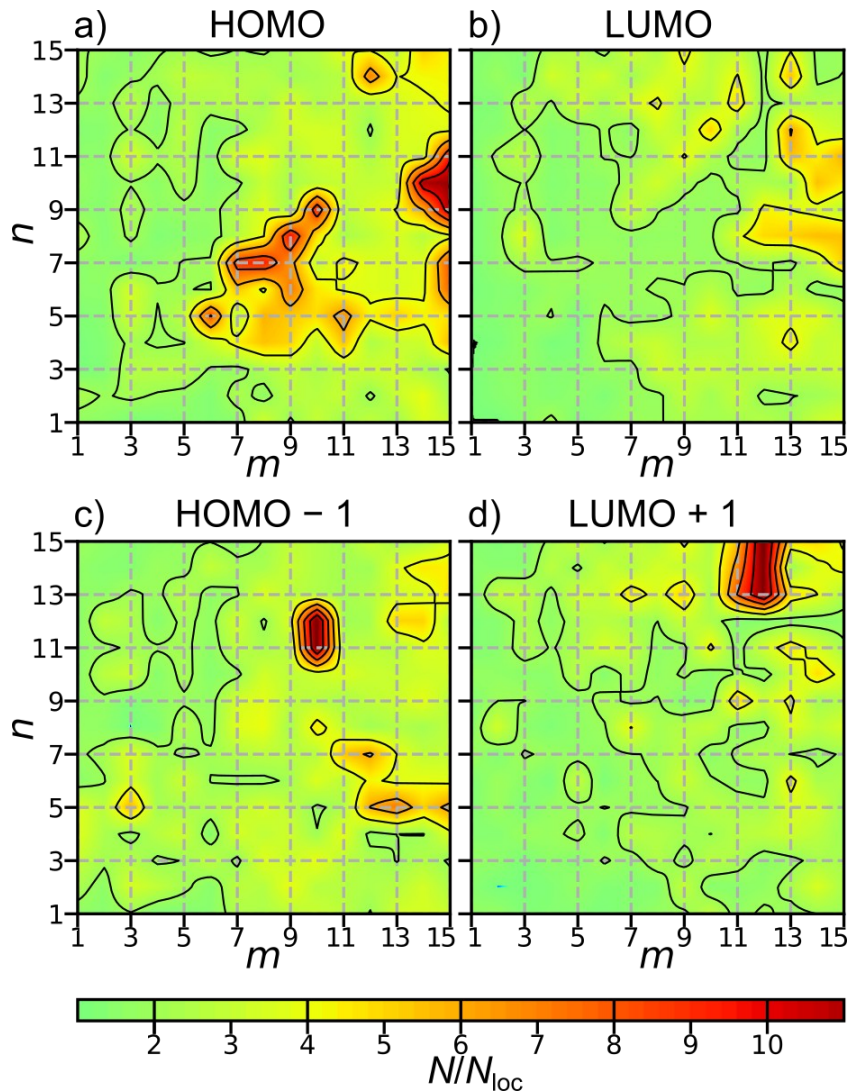
close structures

close properties





# Maps of the localizations in CdnSem clusters: The higher $N/N_{loc}$ – the more localized state

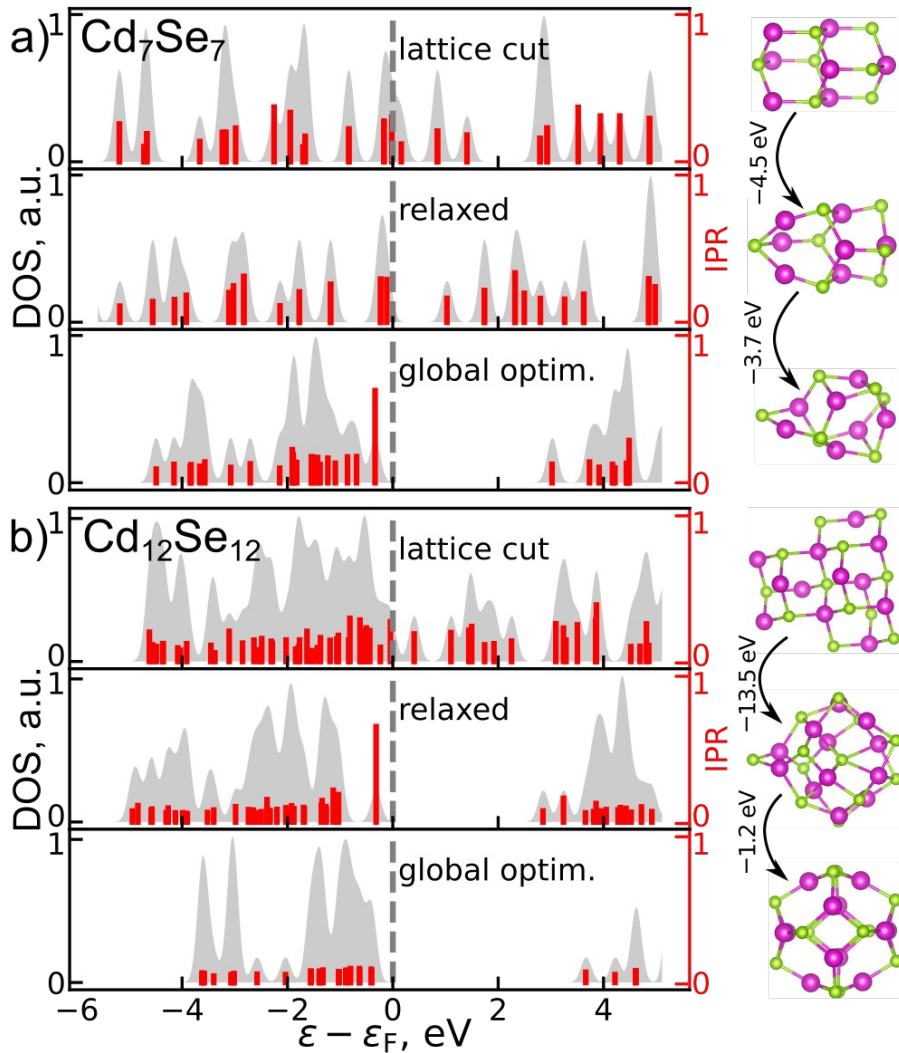


3. Despite the absence of passivating ligands, most clusters are **trap-free**.

Contrary to the previous studies where traps were thought to be inherent to unpassivated crystalline surface.

The reason is the **self-healing effect** due to passivation

# Self-healing of traps



Three stages of stabilization:

1 Cluster is simply cut from a zincblende:

(near-zero gap)

2. Same cluster, relaxed

(opening the gap, mid-gap states are suppressed)

3. Global optimum structure

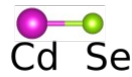
(gap is wide, no mid-gap states)

# Types and mechanisms of trapping

## Trap Symbol

Schematic structure of a trap

Example: cluster & trap orbital



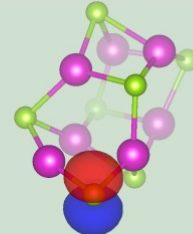
$\langle N_{loc} \rangle$  - localization (atoms)

$N_{tr}$  - abundance

### $T^I_a(\text{Se } p_z)$

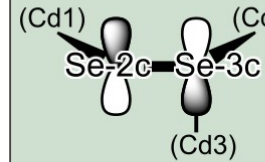


$\langle N_{loc} \rangle = 1.9, N_{tr} = 11$

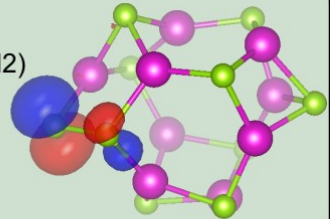


$\text{Cd}_7\text{Se}_7$  (HOMO)

### $T^I_b(\text{Se } \pi^*)$

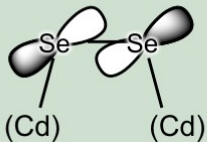


$\langle N_{loc} \rangle = 2.3, N_{tr} = 9$

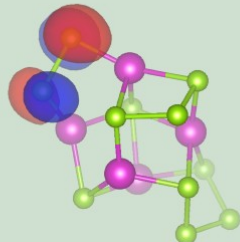


$\text{Cd}_9\text{Se}_{10}$  (HOMO)

### $T^I_c(\text{Se } \pi^*)$

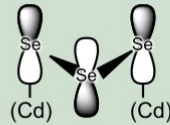


$\langle N_{loc} \rangle = 2.4, N_{tr} = 1$

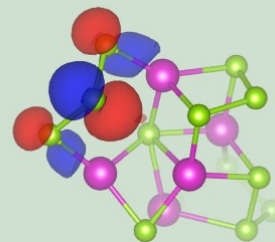


$\text{Cd}_5\text{Se}_{11}$  (HOMO)

### $T^I_d(\text{Se } \pi^*)$

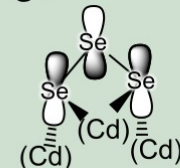


$\langle N_{loc} \rangle = 3.1, N_{tr} = 1$

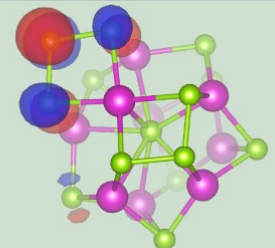


$\text{Cd}_5\text{Se}_{13}$  (HOMO)

### $T^I_e(\text{Se } \pi^*)$

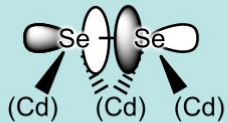


$\langle N_{loc} \rangle = 3.8, N_{tr} = 1$

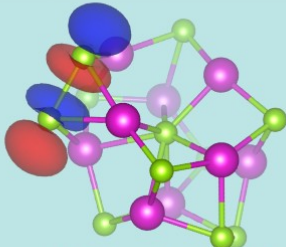


$\text{Cd}_9\text{Se}_{14}$  (HOMO)

### $T^{II}_a(\text{Se } \sigma^*)$

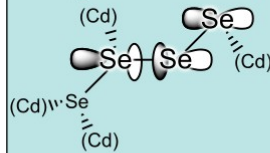


$\langle N_{loc} \rangle = 3.9, N_{tr} = 1$

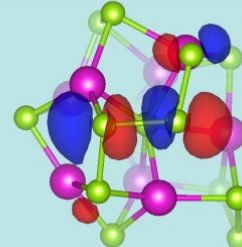


$\text{Cd}_9\text{Se}_{11}$  (LUMO+1)

### $T^{II}_b(\text{Se } \sigma^*)$

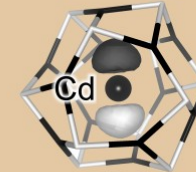


$\langle N_{loc} \rangle = 3.9, N_{tr} = 1$

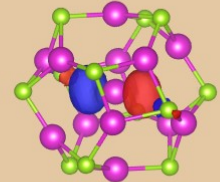


$\text{Cd}_8\text{Se}_{14}$  (LUMO)

### $T^{III}(\text{Cd}@NC)$

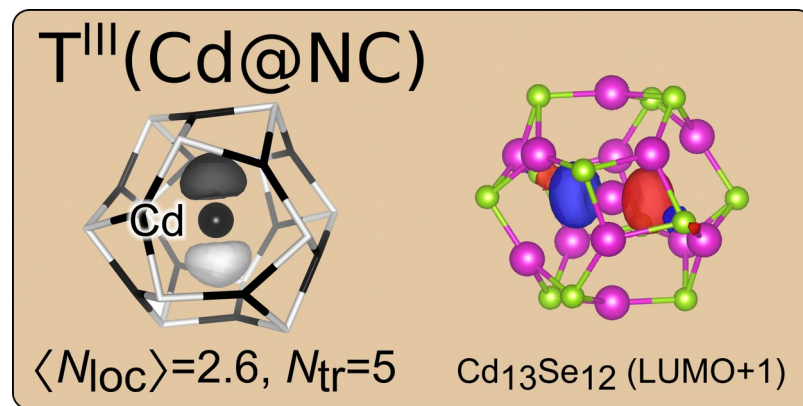
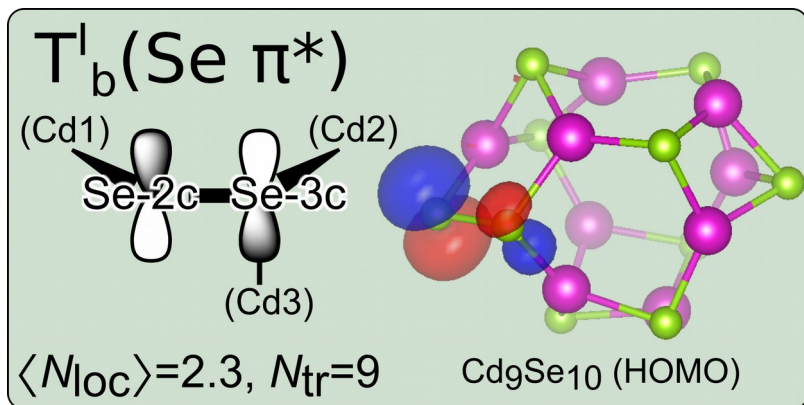


$\langle N_{loc} \rangle = 2.6, N_{tr} = 5$



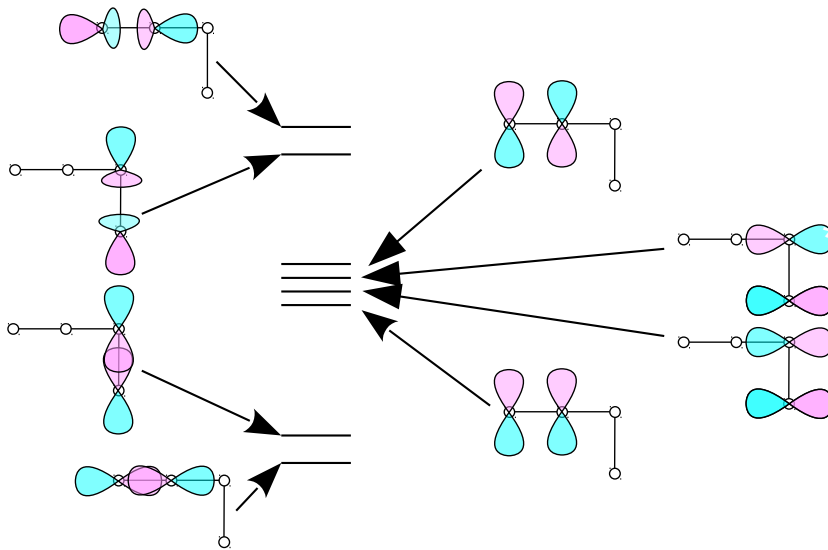
$\text{Cd}_{13}\text{Se}_{12}$  (LUMO+1)

# New mechanisms





# New mechanisms: $T_b^I(\text{Se } \pi^*)$



$\pi$ - and  $\sigma$ -bonding have different energies ( $h_\sigma \gg h_\pi$ )

$$\psi_{\pi^*}(\mathbf{r}) \sim \exp\left\{-d_\sigma \left| (h_\sigma - h_\pi) 2m/\hbar^2 \right|^{1/2}\right\}$$

# Conclusions (CdSe)

1. Electron localization in  $\text{Cd}_n\text{Se}_m$  is inspected in a wide area of compositions ( $n, m = 1, \dots, 15$ )
2. The most widespread traps were found in an unbiased way and classified into three types
3. The corresponding confinement mechanisms are explained at the atomistic level
3. The effect of localization on the HOMO-LUMO gap is investigated

