# Ruby, metals, and MgO as alternative pressure scales: A semiempirical description of shockwave, ultrasonic, $x$-ray, and thermochemical data at high temperatures and pressures 

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#### Abstract

We have constructed semiempirical equations of state of $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}$, and W , which within experimental error bars describe the available shock-wave, ultrasonic, x-ray, and thermochemical data in the temperature range from $10-20 \mathrm{~K}$ up to the melting temperature and to compression $x=V / V_{0}=0.5-0.7$. The comparison of the calculated room-temperature isotherms for these metals with quasihydrostatic measurements supports recently proposed ruby pressure scales. We recommend a new ruby pressure scale in the form $P$ $=A\left(\Delta \lambda / \lambda_{0}\right) \times\left(1+m \Delta \lambda / \lambda_{0}\right)$ with parameters $A=1884 \mathrm{GPa}$ and $m=5.5$. The cross check on independent data confirms the obtained $P V T$ equations of state of $\mathrm{Ag}, \mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}, \mathrm{W}, \mathrm{MgO}$, and diamond. The equations of state of these materials obtained here provide accurate and versatile means for calibrating pressure at all temperatures below the melting point. Furthermore, they can be used for accurate tabulation of thermodynamic properties (heat capacities, entropies) of these reference substances in a wide P-T range.


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## I. INTRODUCTION

More than a decade after the invention of diamond anvil cell (DAC) (Ref. 1), in the beginning of 1970s, the optical ruby fluorescence method of pressure measurement was developed. ${ }^{2}$ With the efforts of many researchers ${ }^{2-6}$ in 1970s and 1980s the ruby pressure scale was calibrated up to megabar pressures and became the de facto standard for pressure measurements in the DAC in the end of the twentieth and beginning of the twenty-first centuries (see reviews in Refs. 7-9).

With the ruby pressure scale, pressure is obtained from the $R_{1}$ line shift of ruby luminescence. This is a secondary pressure scale, requiring a careful calibration. The most popular calibration due to Mao et al. ${ }^{6}$ is based on the measurements of the $R_{1}$ line shift of ruby luminescence in the Ar pressure medium up to a pressure of 80 GPa . The pressure was determined from the room-temperature isotherms of Cu and Ag reduced by Carter et al. ${ }^{10}$ from shock-wave data. The value for the initial slope of $P(\lambda), A=\lambda(\mathrm{d} P / \mathrm{d} \lambda)=1904 \mathrm{GPa}$, has been obtained by Piermarini et al. ${ }^{3}$ with the use of Decker's ${ }^{11}$ equation of state (EOS) of NaCl as pressure standard in the quasihydrostatic medium (up to 10.4 GPa ) and in the nonhydrostatic medium up to the pressure of 19.5 GPa . The resulting pressure scale of Mao et al. ${ }^{6}$ is expressed as

$$
\begin{gather*}
P=\frac{A}{B}\left[\left(1+\Delta \lambda / \lambda_{0}\right)^{B}-1\right], \\
P=\frac{A}{B}\left[\left(\lambda / \lambda_{0}\right)^{B}-1\right], \tag{1a}
\end{gather*}
$$

where $P$ is pressure in $\mathrm{GPa}, A=\lambda(\partial P / \partial \lambda)=1904 \mathrm{GPa}, B$ $=7.665, \lambda_{0}=694.24 \mathrm{~nm}$.

Almost at the same time, Aleksandrov et al. ${ }^{12}$ published a significantly different calibration of the ruby scale based on the a priori EOS of diamond. Aleksandrov et al. ${ }^{12}$ performed simultaneous measurements of the $R_{1}$ line shift of ruby luminescence and spectra of the first-order Raman light scat-

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tering of diamond in a DAC with helium pressuretransmitting medium up to the compression of $x=V / V_{0}$ $=0.93$. Trying various pressure scales, Aleksandrov et al. ${ }^{12}$ obtained unrealistically low values of the pressure derivatives of the bulk modulus ( $K^{\prime}=d K / d P$ ) for diamond: $1,1.9$, and 2.5 for scales from Refs. 3, 5, and 8, respectively, in conflict with theory, ${ }^{13,14}$ and ultrasonic measurements. ${ }^{15}$ Assuming a much more realistic value of $K^{\prime}=4$ for diamond, Aleksandrov et al. ${ }^{12}$ arrived at a new pressure scale with parameters $A=1918 \mathrm{GPa}$ and $B=11.7$ for Eq. (1). Aleksandrov et al. ${ }^{12}$ also proposed another form for the pressure scale:

$$
\begin{equation*}
P=A\left(\Delta \lambda / \lambda_{0}\right)\left(1+m \Delta \lambda / \lambda_{0}\right) \tag{2}
\end{equation*}
$$

with parameters $A=1892 \pm 13 \mathrm{GPa}$ and $m=6.4$. This scale begins to differ from the scale of Ref. 6 above 20 GPa , and leads to significant differences at pressures greater than 50 GPa (Fig. 1).

Hemley et al. ${ }^{16}$ have obtained the EOS of solid neon up to 110 GPa using the ruby ${ }^{6}$ and tungsten ${ }^{17}$ pressure scales and have overall confirmed the ruby scale of Mao et al. ${ }^{6}$ However, they noticed that pressures from the tungsten scale are systematically higher (5\%) than those from the ruby scale at high compression, but this difference was within the expected error in the tungsten isotherm (Ref. 16, p. 11822). Helium remains considerably weaker than neon and argon at high pressure, ${ }^{18}$ therefore Hemley et al. ${ }^{16}$ suggested that part of the differences between the ruby scales of Aleksandrov et $a l .{ }^{12}$ and Mao et al. ${ }^{6}$ may be associated with residual nonhydrostatic effects in the Ar pressure medium used in Ref. 6.

Recently, Zha et al. ${ }^{19}$ measured the elastic constants of MgO up to a pressure of 55 GPa using Brillouin scattering. By combining the Brillouin and x-ray measurements, they obtained the equation of MgO state and calculated a new ruby scale in the form (1) with $A=1904 \mathrm{GPa}$ and $B=7.715$, close to the calibration of Mao et al. ${ }^{6}$

However, a series of works ${ }^{20-24}$ were recently published, in which the ruby pressure scale of Mao et al. ${ }^{6}$ has been


FIG. 1. Comparison of early calibrations of the ruby pressure scale. The linear relation comes from Piermarini et al. (Ref. 3). The nonlinear functions were calculated with Eq. (1): $A=1904 \mathrm{GPa}$ and $B=5$ (Mao et al. (Ref. 5), nonhydrostatic conditions), $A$ $=1904 \mathrm{GPa}$ and $B=7.665$ (Mao et al. (Ref. 6), quasihydrostatic conditions, argon pressure transmitting medium), $A=1918 \mathrm{GPa}$ and $B=11.5$ (Aleksandrov et al. (Ref. 12) quasihydrostatic conditions, helium pressure transmitting medium).
considerably shifted toward the scale of Aleksandrov et al. ${ }^{12}$
The scale of Holzapfel ${ }^{20}$ is based on a comparison of recent high-pressure x-ray diffraction data for diamond and Ta with low-pressure ultrasonic data. It shows a significant difference from the scales of Mao et al. ${ }^{5,6}$ and comes close to the scale of Aleksandrov et al. ${ }^{12}$ It has the following form:

$$
\begin{equation*}
P=\frac{A}{B+C}\left[\exp \left(\frac{B+C}{C}\left[1-\left(\lambda / \lambda_{0}\right)^{-C}\right]\right)-1\right], \tag{3}
\end{equation*}
$$

where $A=1820 \mathrm{GPa}, B=14, C=7.3$, and corresponds to Eq. (1) in the limit of $C \rightarrow 0$.

Dorogokupets and Oganov ${ }^{21}$ have constructed the EOSs of Cu and Ag , which agree with experimental measurements in the temperature range from $10-20 \mathrm{~K}$ up to the melting temperature and up to the compression $x=0.6$, and have obtained a ruby pressure scale with parameters $A=1871 \mathrm{GPa}$ and $B=10.06$ for Eq. (1). This scale agrees quite well with the Holzapfel scale ${ }^{20}$ up to 100 GPa .

Kunc et al. ${ }^{22,23}$ have compared the theoretical EOS of diamond with high-quality data of Occelli et al. ${ }^{25}$ on x-ray diffraction of diamond in DAC with helium pressuretransmitting medium. Using the ruby scale of Mao et al., ${ }^{6}$ Occelli et al. ${ }^{25}$ obtained the value $K^{\prime}=3.0 \pm 0.1$, and this value appeared too low in comparison with the value from ultrasonic measurements $K^{\prime}=4 \pm 0.5$ (Ref. 15) and theoretical value $K^{\prime}=3.65 \pm 0.05$ of Kunc et al. ${ }^{22,23}$ Kunc et al. ${ }^{22,23}$ have assumed that the discrepancy between experimental and calculated EOS and phonon frequencies of diamond is caused by some error in the Mao et al. ${ }^{6}$ ruby pressure scale. To remove the discrepancy for the phonon frequency, Kunc et $a l .{ }^{22,23}$ have proposed a revised ruby scale in the form

$$
\begin{equation*}
P=A(\Delta \lambda / \lambda)(1+\mu \Delta \lambda / \lambda) \tag{4}
\end{equation*}
$$

with parameters $A=1860 \mathrm{GPa}$ and $\mu=7.75$.

For calibration of the ruby pressure scale, it is necessary to recognize as revolutionary the work of Dewaele et al., ${ }^{24,26}$ where $P V$ relations at room temperature have been measured for $\mathrm{Ta}, \mathrm{Au}$, and Pt to 94 GPa and for $\mathrm{Al}, \mathrm{Cu}$, and W to 153 GPa in DAC with helium pressure-transmitting medium. When they compared the EOSs of these metals based on the ruby scale of Mao et al. ${ }^{6}$ and the room-temperature isotherms obtained by reducing shock-wave data ${ }^{27-29}$ it appeared that they differ by up to $8-9 \mathrm{GPa}$ at pressures of $100-150 \mathrm{GPa}$. Correcting the ruby pressure scale for these differences, Dewaele et al. ${ }^{24}$ have obtained a new pressure scale with parameters $A=1904 \mathrm{GPa}$ and $B=9.5$ for Eq. (1). This scale gives practically the same pressures as scales of Holzapfel ${ }^{20}$ and of Dorogokupets and Oganov. ${ }^{21}$

Here we present a general thermodynamic formalism, using which we construct semiempirical EOSs of several reference substances ( $\mathrm{Ag}, \mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}, \mathrm{W}$, diamond, MgO ) in the temperature range $10-15 \mathrm{~K}$ to the melting temperature and up to pressures $200-300 \mathrm{GPa}$ (compression $x$ $=V / V_{0}=0.5-0.7$ ). We show that these EOSs accurately describe experimental measurements of the heat capacity, thermal expansion, and adiabatic bulk modulus at 1 atm and the data from the Shock Wave Database [SWDB (Ref. 30)]. Comparison of our calculated room-temperature isotherms with quasihydrostatic data of Dewaele et al. ${ }^{24,26}$ allowed us to obtain a new ruby pressure scale. Based on the newly obtained ruby pressure scale, we explored different functional forms for $P(\lambda)$, and found that the form (1) is valid for pressures below 100 GPa . Cross checks between the obtained EOSs of $\mathrm{Ag}, \mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}, \mathrm{W}, \mathrm{MgO}$ and independent data further validate our results and show that the obtained EOSs of all studied substances are consistent with each other on different isotherms and with the ruby pressure scale on room isotherm. As a basis of construction of the EOS we will use the expanded formalism from Ref. 21.

## II. THERMODYNAMICS

Let us write the Helmholtz free energy $F(V, T)$ as the sum ${ }^{31}$
$F=U_{0}+E(V)+F_{\mathrm{qh}}(V, T)+F_{\mathrm{anh}}(V, T)+F_{\mathrm{el}}(V, T)+F_{\mathrm{def}}(V, T)$,
where $U_{0}$ is the reference energy, $E(V)$ is the potential (cold) part of the free energy on the reference isotherm, which depends only on volume; $F_{\text {qh }}(V, T), F_{\text {anh }}(V, T), F_{\text {el }}(V, T)$, and $F_{\text {def }}(V, T)$ are the quasiharmonic part of the Helmholtz free energy, and terms describing intrinsic anharmonicity, electronic contribution, and thermal defects.

Differentiating (5), we obtain all the necessary thermodynamic functions: entropy, $S=-(\partial F / \partial T)_{V}$, internal energy $E$ $=F+T S$, heat capacity at constant volume, $C_{V}=(\partial E / \partial T)_{V}$, pressure, $P=-(\partial F / \partial V)_{T}$, isothermal bulk modulus, $K_{T}$ $=-V(\partial P / \partial V)_{T}$, slope of pressure at constant volume $(\partial P / \partial T)_{V}=\alpha K_{T}$, where $\alpha=1 / V(\partial V / \partial T)_{P}$. Heat capacity at constant pressure is $C_{P}=C_{V}+\alpha^{2} T V K_{T}$, adiabatic bulk modulus is $K_{S}=K_{T}+V T\left(\alpha K_{T}\right)^{2} / C_{V}$. The enthalpy and Gibbs energy can be found from $H=E+P V, G=F+P V$.

Cold energy, pressure, and bulk modulus are written as ${ }^{32}$

$$
\begin{gather*}
E(V)=9 K_{0} V_{0} \eta^{-2}\{1-[1-\eta(1-y)] \exp [(1-y) \eta]\},  \tag{6}\\
P(V)=-\partial E / \partial V=3 K_{0} y^{-2}(1-y) \exp [(1-y) \eta]  \tag{6a}\\
K(V)=K_{0} y^{-2}[1+(\eta y+1)(1-y)] \exp [(1-y) \eta] \tag{6b}
\end{gather*}
$$

where $y=x^{1 / 3}=\left(V / V_{0}\right)^{1 / 3}$ and $\eta=1.5\left(K^{\prime}-1\right), K^{\prime}=d K / d P$, $V_{0}$, and $K_{0}$ are molar volume and bulk modulus at reference conditions ( $T_{0}=298.15 \mathrm{~K}, P_{0}=1 \mathrm{bar}$ ).

For approximation of the quasiharmonic phonon part of the Helmholtz free energy through the whole range of temperatures we use a modified formalism of Kut'in et al.: ${ }^{33,34}$

$$
\begin{equation*}
F_{\mathrm{qh}}=m_{B} \mathrm{R}\left[\frac{(d-1)}{2 d} \Theta_{B}-T \ln (1+b)\right], \tag{7}
\end{equation*}
$$

where $R$ is the gas constant, $m_{B}$ is the number of phonon modes; $b=1 /[\exp (g)-1], g=d \ln \left[1+\Theta_{B} /(T d)\right], d$ is the exponential parameter controlling the behavior of the lowtemperature limiting behavior of the heat capacity, and $\Theta_{B}$ is the characteristic temperature. These authors noticed that this analytical function yields both a $C_{V} \sim T^{d}$ dependence of the heat capacity at low temperatures and constant limit $C_{V}$ $=m_{B} R$ at high temperatures. Kut' in et al. ${ }^{33,34}$ have shown that Eq. (7) in combination with the Einstein function approximates well thermodynamic functions from 0 K up to the ambient temperature and higher. This formalism can be used for analytical representation of the quasiharmonic part of the Helmholtz free energy, which can be written as

$$
\begin{align*}
F_{\mathrm{qh}}= & \sum_{i} m_{B i} \mathrm{R}\left[\frac{\left(d_{i}-1\right)}{2 d_{i}} \Theta_{B i}-T \ln \left(1+b_{i}\right)\right] \\
& +\sum_{j} m_{E j} \mathrm{R}\left[\frac{\Theta_{E j}}{2}+T \ln \left(1-\exp \frac{-\Theta_{E j}}{T}\right)\right] \tag{8}
\end{align*}
$$

where $\Theta_{B i}$ and $\Theta_{E j}$ are the Bose-Einstein and the Einstein characteristic temperatures, which depend on volume (or $x$ $=V / V_{0}$ ). Usually for a very accurate approximation it is enough to take two Bose-Einstein contributions and two Einstein contributions.

For the volume dependence of the Grüneisen parameter we used the Al'tshuler et al. ${ }^{35}$ form:

$$
\begin{equation*}
\gamma=\gamma_{\infty}+\left(\gamma_{0}-\gamma_{\infty}\right)\left(V / V_{0}\right)^{\beta}=\gamma_{\infty}+\left(\gamma_{0}-\gamma_{\infty}\right) x^{\beta} \tag{9}
\end{equation*}
$$

where $\gamma_{0}$ is the Grüneisen parameter at ambient conditions, $\gamma_{\infty}$ is the Grüneisen parameter at infinite compression ( $x$ $=0$ ), and $\beta$ is a fitted parameter. The form (9) is simple and convenient, has a correct behavior at infinite compression $(\gamma \rightarrow$ constant, $q \rightarrow 0)$ and in our experience describes extremely well results of theoretical calculations. From (9) it is possible to calculate the volume dependence of each characteristic temperature (here we use the same $\gamma$ for all frequencies) and parameter $q$ :

$$
\begin{gather*}
\Theta=\Theta_{0} x^{-\gamma_{\infty}} \exp \left[\frac{\gamma_{0}-\gamma_{\infty}}{\beta}\left(1-x^{\beta}\right)\right],  \tag{10}\\
q=d \ln \gamma / d \ln V=\beta x^{\beta} \frac{\gamma_{0}-\gamma_{\infty}}{\gamma} \tag{11}
\end{gather*}
$$

We describe the contribution of intrinsic anharmonicity to the Helmholtz free energy using the formulation of Oganov and Dorogokupets: ${ }^{36}$

$$
\begin{align*}
F_{\text {anh }}= & \sum_{i, j} m_{i, j} \mathrm{R} \frac{a x^{m}}{6} \times\left[\left(\frac{1}{2} \Theta_{i, j}+\frac{\Theta_{i, j}}{e^{\Theta_{i, j} / T}-1}\right)^{2}\right. \\
& \left.+2\left(\frac{\Theta_{i, j}}{T}\right)^{2} \frac{e^{\Theta_{i, j} / T}}{\left(e^{\Theta_{i, j} / T}-1\right)^{2}} \times T^{2}\right] \tag{12}
\end{align*}
$$

where indices $i$ and $j$ denote Bose-Einstein and Einstein terms, respectively. Equation (12) was obtained in the first order of thermodynamic perturbation theory and has correct high- and low-temperature behavior, and contains a contribution of zero-point anharmonic effects.

The electronic component of the Helmholtz free energy is taken as

$$
\begin{equation*}
F_{\mathrm{el}}=-\frac{3}{2} n \mathrm{Re} e x^{g} T^{2} \tag{13}
\end{equation*}
$$

where we assume the free-electron value $g=2 / 3$ for $\mathrm{Cu}, \mathrm{Ag}$, and Au. ${ }^{31}$

For the contribution of thermal defects we use the commonly accepted approximation of independent monovacancies: ${ }^{37}$

$$
\begin{equation*}
F_{\mathrm{def}}=-\frac{3}{2} n \mathrm{R} T \exp \left(S x^{f}-\frac{H x^{h}}{T}\right) \tag{14}
\end{equation*}
$$

where $S$ and $H$ are the entropy and enthalpy of formation of a monovacancy, respectively, and for all metals we assumed $f=-1, h=-2$.

Pressure on the shock-wave adiabats was calculated as follows: ${ }^{31}$

$$
\begin{equation*}
P_{H}=\frac{P(V)-\frac{\gamma}{V}\left[E(V)-E_{0}\right]}{1-\frac{\gamma(1-x)}{2 x}} . \tag{15}
\end{equation*}
$$

Our procedure for finding the parameters describing thermodynamics and EOSs consists of two stages. In the first stage, we find preliminary values of the parameters using weighted least-squares fitting to experimental data unbiased by pressure calibration (i.e., measurements at 1 atm and shock-wave data). It starts with some reasonable guesses for $V_{0}, K_{0}, K^{\prime}$, and $\gamma_{0}$. Low-temperature measurements of the heat capacity (up to $300-350 \mathrm{~K}$ ) lead to the determination of the characteristic temperatures $\left(\Theta_{B 1}, \Theta_{B 2}, \Theta_{E 1}, \Theta_{E 2}\right)$ and $d_{1}, d_{2}, m_{B 1}, m_{B 2}, m_{E 1}, m_{E 2}$ parameters (with the constraint that the total number of modes $m_{B i}$ and $m_{E j}$ must be equal $3 n$, where $n$ is the number of atoms). The starting value of the anharmonicity parameter $a$ is estimated from hightemperature measurements of the heat capacity or relative enthalpy. With this starting set of parameters, we optimize all the parameters by simultaneous fitting of the available experimental data on the heat capacity and relative enthalpy, volume, thermal expansion, and adiabatic bulk modulus at zero pressure and various temperatures, and Hugoniot pressure at $x$. The full solution allowed us to find all the necessary parameters: $V_{0}, K_{0}, K^{\prime}, \Theta_{B 1}, \Theta_{B 2}, \Theta_{E 1}, \Theta_{E 2}, d_{1}, d_{2}, m_{B 1}$,
$m_{B 2}, m_{E 1}, m_{E 2}, \gamma_{0}, \gamma_{\infty}, \beta, a, m, e, H, S$. Some of these parameters, however, could in principle be specified a priori, e.g., from accurate theoretical calculations or from other sources. For example, from the measurements of the heat capacity up to 30 K it is possible to estimate the electronic contribution to the heat capacities [parameter $e$ in Eq. (13)]. Fitted values of $e$ (Table I) do not exceed the values obtained from the experimental measurements of heat capacity. For the contribution of thermal defects, parameters $f$ and $h$ determining volumetric dependence of the defect enthalpy and entropy were fixed. The contribution of thermal defects to the Helmholtz free energy can be estimated also from various experimental data (see, for example, Ref. 38).

In the second stage we also include static compression data calibrated with different pressure scales, refit all the parameters and check their consistency with ultrasound measurements of $K_{S}$ as a function of temperature at 1 atm . With the above formulation we can carry out a simultaneous processing of all the available measurements of the heat capacity, thermal expansion coefficient, volume, and adiabatic and isothermal bulk moduli at zero pressure, static measurements of volume on a room-temperature isotherm and at higher temperatures, shock-wave data, and calculate any thermodynamic functions vs $T$ and $V$ or vs $T$ and $P$. In practical realization, we write the Helmholtz free energy relative to the reference conditions $T_{0}=298.15 \mathrm{~K}$ and $P_{0}=1$ bar; consequently, the fitted parameters that we find correspond to ambient conditions. At those conditions it is usually easy to verify our parameters by performing direct measurements.

## III. RESULTS

First, we obtained all the fitted parameters from a simultaneous processing of experimental data for $C_{P}, \alpha, V, K_{S}$ at zero pressure and shock-wave data. Originally such analysis has been carried out for $\mathrm{Al}, \mathrm{Cu}, \mathrm{Ta}$, and W , for which the values of $K_{0}$ and $K^{\prime}$ equal to 73.46 GPa and 4.52 , 133.98 GPa and $5.41,191.44 \mathrm{GPa}$ and $3.93,306.23 \mathrm{GPa}$ and 4.16 at ambient conditions have been obtained, and this results in a catastrophic divergence from the data of Dewaele et al. ${ }^{24}$ above 30 GPa if the Mao et al. ${ }^{6}$ ruby pressure scale is used for calibrating pressure for experimental data of Dewaele et al. ${ }^{24}$ - see Fig. 2(a).

From the EOSs of these metals a preliminary ruby scale with parameters $A=1885 \mathrm{GPa}$ and $B=10.4$ for Eq. (1) was obtained. Parameter $A$ has been fixed as the average of Piermarini et al. ${ }^{3}$ data for quasihydrostatic conditions (A $=1896 \mathrm{GPa}$, also used in Ref. 21) and direct determination $A=1875 \pm 30 \mathrm{GPa} .{ }^{39}$ Nakano et al. ${ }^{40}$ at 10 K and pressures $0-22 \mathrm{GPa}$ found $\mathrm{d} P / \mathrm{d} \lambda=2.748 \pm 0.012 \mathrm{GPa} / \mathrm{nm}$, from which $A=1908 \pm 8 \mathrm{GPa}$, close to the result of Ref. 3. However, Holzapfel ${ }^{20}$ and Syassen ${ }^{41}$ commented that the initial slope $\mathrm{d} P / \mathrm{d} \lambda$ should be smaller than the results of Refs. 3 and 40.

Using this pressure scale for calibration of the Dewaele et $a l .{ }^{24}$ measurements, the EOSs for Au and Pt have been obtained. The differences from Dewaele et al. ${ }^{24}$ pressure calculated using the ruby pressure scale with parameters $A$ $=1885 \mathrm{GPa}$ and $B=10.4$ for Eq. (1) are shown in Fig. 2(b).


FIG. 2. (a) Pressure difference for the Dewaele et al. (Refs. 24 and 26) measurements calibrated using the Mao et al. (Ref. 6) ruby scale and our calculated room-temperature isotherms for $\mathrm{Cu}, \mathrm{Al}, \mathrm{Ta}$, and W. (b) Difference between the Dewaele et al. (Refs. 24 and 26) pressure using the ruby scale with parameters $A=1885 \mathrm{GPa}$ and $B$ $=10.4$ for Eq. (1) and our calculated room-temperature isotherms for $\mathrm{Cu}, \mathrm{Al}, \mathrm{Ta}, \mathrm{Pt}, \mathrm{Au}$, and W .

Though the differences are small (within 2 GPa , i.e., $2-3 \%)$, Fig. 2(b) shows a minimum of absolute deviations in the pressure range $70-120 \mathrm{GPa}$. If one assumes that the experimental measurements of Dewaele et al. ${ }^{24}$ are absolutely reliable, then it is necessary to continue the analysis.

We consider two possible causes of this minimum:
(1) phase transitions of ruby at these pressures, ${ }^{42}$
(2) functional dependence (1) is not suitable for highpressure extrapolation.

We have carried out an additional analysis of the EOSs of all six metals studied in Ref. 24 up to 100 GPa (fitted parameters are listed in Table I), varying parameter $B$ in Eq. (1) and recalculating all the EOSs until consistency with all the experimental measurements of thermodynamic functions at zero pressure $\left(C_{P}, \alpha, K_{S}\right)$ and measurements of pressure at given volume.

As a result we have obtained a new ruby pressure scale with parameters $A=1885 \mathrm{GPa}$ and $B=11$ for Eq. (1), which agrees very well with the data of Dewaele et al. ${ }^{24}$ up to the pressure of 100 GPa (Fig. 3). However, above 100 GPa we have systematic deviations from the measurements of Dewaele et al. ${ }^{24}$ If this is due to a phase transition, then we need two ruby pressure scales: one with $A=1885 \mathrm{GPa}$ and $B=11$ for Eq. (1) up to the pressure 85 GPa , the other with $A$ $=1975 \mathrm{GPa}$ and $B=8.59$ at pressures above 85 GPa . However, it is likely that at room temperature ruby will keep its structure in the metastable state even at ultrahigh pressures,


FIG. 3. Difference between the Dewaele et al. (Refs. 24 and 26) pressure using the ruby scale with parameters $A=1885 \mathrm{GPa}$ and $B$ $=11$ for Eq. (1) and our calculated room-temperature isotherms for $\mathrm{Cu}, \mathrm{Al}, \mathrm{Ta}, \mathrm{Pt}, \mathrm{Au}$ and W. From here on we show pressures according to our present scale (16).
and thus a search for an alternative functional form is needed for the ruby pressure scale.

Among alternative $P(\lambda)$ functional forms, one could use the Holzapfel ${ }^{20}$ three-parametrical equation in the form (3), however, we find that better results are obtained with the two-parametrical equation in the Aleksandrov et al. ${ }^{12}$ form:

$$
\begin{equation*}
P=1884 \times\left(\Delta \lambda / \lambda_{0}\right) \times\left(1+5.5 \Delta \lambda / \lambda_{0}\right) \tag{16}
\end{equation*}
$$

Now the deviations of the calculated room-temperature isotherms from the data of Dewaele et al. ${ }^{24,26}$ are very small, only occasionally slightly in excess of 1 GPa at pressures up to 160 GPa (Fig. 4). Figure 5 shows recent calibrations of the ruby pressure scale in comparison with previous calibrations of Mao et al. ${ }^{6}$ and Aleksandrov et al. ${ }^{12}$ and recalculated data of Dewaele et al. ${ }^{24,26}$ for $\mathrm{Al}, \mathrm{Cu}$, and W using the ruby pressure scale in the form (16), at pressures in the range 100-155 GPa.

As just described, we have obtained two variants of the ruby pressure scale with different functional dependences $P(\lambda)$ which give identical pressure up to 155 GPa . As the final version we prefer Eq. (16) which has the form proposed by Aleksandrov et al. ${ }^{12}$ The final parameters of the EOSs of the reference materials considered here are given in Table I.


FIG. 4. Difference between the Dewaele et al. (Refs. 24 and 26) pressure using the ruby pressure scale (16) and our calculated roomtemperature isotherms for $\mathrm{Cu}, \mathrm{Al}, \mathrm{Ta}, \mathrm{Pt}, \mathrm{Au}$, and W .


FIG. 5. Comparison of recent ruby pressure scales [Kunc et al. (Refs. 22, 23, and 41) Chijioke et al. (Ref. 43), Holzapfel, (Ref. 44) and Eq. (16)] at pressures $100-155 \mathrm{GPa}$ with previous calibrations of Mao et al. (Ref. 6) and Aleksandrov et al. (Ref. 12). Black rectangles are pressures calculated using the ruby scales of Refs. 20, 21, and 24 (they nearly coincide on the scale of this figure). Also shown are pressures calibrated from the EOSs of $\mathrm{Cu}, \mathrm{Al}$, and W obtained here.

We conclude that the EOSs of $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}$, and W obtained here agree with experimental measurements in the temperature and pressure ranges considered here with an error comparable to that of direct measurements (comparison of the calculated and experimental thermodynamic functions is shown in Fig. 6 for the case of tungsten and in Dorogokupets and Oganov ${ }^{45}$ for the case of gold). One can expect that the obtained EOSs will be also correct at higher temperatures and pressures, which will be confirmed through the comparison of the EOSs of gold, platinum, silver, and MgO .

A critical role in our approach is played by the ultrasonic measurements of the temperature dependence of the adiabatic bulk modulus in a wide range of temperatures at zero pressure. Comparison of these data with our calculation is shown on Fig. 7, which shows good agreement of our calculation with ultrasonic data.

Figure 7 shows the adiabatic bulk modulus of Au calculated by Shim et al., ${ }^{73}$ which differs significantly from experiment and our calculation. The main cause of this discrepancy is the value $K^{\prime}=5$ recommended in Ref. 73 for gold ( $K^{\prime}=5$ comes from Ref. 89, where the pressure scale of Zha et al. ${ }^{19}$ was used), which is significantly lower than results of Table I and Ref. 66. The second reason is that the MieGrüneisen formalism, used in Ref. 73 and valid in the framework of the quasiharmonic approximation, does not allow one to describe well the high-temperature experimental data.

For Pt the temperature dependence of the adiabatic bulk modulus was determined in Ref. 74, but the results appeared to be internally inconsistent. The filled squares are values $K_{S}$ $(T)$ from Fig. 1 in Ref. 74 (these were used by us), and empty squares-computed from the polynomial from Table 1 in Ref. 74. For gold (Ref. 72) such a contradiction did not appear.

Table I also shows the parameters of the EOSs of Ag, diamond, and MgO . The EOS of Ag is an update of our earlier version; ${ }^{21}$ the EOS of periclase will be discussed

TABLE I. Parameters of the proposed EOSs.

| Parameters | Ag | Al | Au | Cu | Pt | Ta | W | MgO | Diamond |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{0}\left(\mathrm{~cm}^{3}\right)$ | 10.272 | 9.999 | 10.215 | 7.113 | 9.091 | 10.851 | 9.545 | 11.248 | 3.417 |
| $K_{0}(\mathrm{GPa})$ | 99.65 | 72.67 | 166.70 | 133.41 | 276.07 | 191.39 | 306.00 | 160.31 | 443.16 |
| $K^{\prime}$ | 6.11 | 4.62 | 6.00 | 5.37 | 5.30 | 3.81 | 4.17 | 4.18 | 3.777 |
| $\Theta_{\mathrm{B} 1}(\mathrm{~K})$ | 130.6 | 245.8 | 95.7 | 123.7 | 95.2 | 72.6 | 182.8 | 447.3 | 1202.1 |
| $d_{\text {B1 }}$ | 8.572 | 5.575 | 8.290 | 3.776 | 8.199 | 5.536 | 13.270 | 11.248 | 9.604 |
| $m_{\text {B1 }}$ | 0.121 | 0.987 | 0.681 | 0.115 | 0.329 | 0.117 | 0.513 | 1.429 | 1.163 |
| $\Theta_{\mathrm{B} 2}(\mathrm{~K})$ | 103.6 | - | 106.4 | 175.4 | 148.4 | 101.8 | 172.5 | 384.0 | 1135.1 |
| $d_{\text {B2 }}$ | 5.326 | - | 3.239 | 10.372 | 4.005 | 24.513 | 3.305 | 3.593 | 3.380 |
| $m_{\text {B2 }}$ | 0.449 | - | 0.417 | 0.711 | 0.383 | 0.396 | 0.174 | 0.276 | 0.218 |
| $\Theta_{\mathrm{E} 1}(\mathrm{~K})$ | 111.9 | 240.2 | 170.6 | 187.4 | 214.6 | 144.0 | 287.6 | 703.8 | 1687.2 |
| $m_{\text {E1 }}$ | 0.766 | 1.000 | 1.063 | 0.756 | 1.211 | 1.118 | 1.166 | 2.570 | 1.396 |
| $\Theta_{\mathrm{E} 2}(\mathrm{~K})$ | 189.12 | 356.2 | 105.2 | 286.9 | 140.8 | 214.9 | 213.8 | 466.0 | 1033.7 |
| $m_{\mathrm{E} 2}$ | 1.664 | 1.013 | 0.839 | 1.418 | 1.077 | 1.369 | 1.145 | 1.725 | 0.223 |
| $\gamma_{0}$ | 2.376 | 2.144 | 2.965 | 1.974 | 2.802 | 1.714 | 1.553 | 1.522 | 0.820 |
| $\gamma_{\infty}$ | 1.481 | 1.017 | 1.142 | 1.554 | 1.538 | 1.241 | 0.694 | 1.111 | 0.615 |
| $\beta$ | 2.507 | 3.942 | 3.030 | 4.647 | 5.550 | 6.825 | 3.698 | 4.509 | 10.121 |
| $a\left(10^{-6} \mathrm{~K}^{-1}\right)$ | 6.70 | 5.14 | 25.33 | 3.50 | 160.9 | 61.9 | -39.3 | 13.56 | -23.85 |
| $m$ | 3.44 | 3.44 | 3.79 | 3.46 | 4.06 | 4.00 | 2.67 | 5.23 | 1.22 |
| $e\left(10^{-6} \mathrm{~K}^{-1}\right)$ | 25.9 | 54.1 | 18.92 | 27.698 | 260.0 | 167.0 | 40.4 | - | - |
| $g$ | 0.666 | $1.8{ }^{\text {a }}$ | 0.66 | 0.666 | $2.4{ }^{\text {a }}$ | $1.3{ }^{\text {a }}$ | $0.2{ }^{\text {a }}$ | - | - |
| $H(\mathrm{~K})$ | 15239 | 8679 | 11.69 | 11687 | 32572 | 36278 | 14714 | - | - |
| $S$ | 0.732 | 0.998 | 1.067 | 1.407 | 0.631 | 4.910 | 0.672 | - | - |

${ }^{\text {a }}$ Reference 31 .
elsewhere, ${ }^{96}$ and the EOS of diamond has been constructed in view of its frequent use for creating pressure scales. The EOS of diamond was based on shock-wave data and experimental measurements of $C_{P}, \alpha, K_{S}$. The obtained value of $K^{\prime}=3.77$ (Table I) turns out to be close to the results of Kunc et al..$^{22,23}$ and coincides with results of $a b$ initio calculations of Oganov (unpublished).

Let us analyze in more detail Fig. 6(b), which shows the differences between the calculated and measured lowtemperature heat capacity of W . These differences for all the considered substances basically do not exceed $1 \%$, therefore the calculated standard entropy of tungsten $S_{298}=32.65 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ coincides with the reference data: $S_{298}=32.64 \pm 0.42 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1} \quad$ (Ref. 97) and $S_{298}$ $=32.66 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ (Ref. 48). For other substances the entropies are (in brackets we give the measured entropies from Ref. 97) Ag 42.72 (42.55 $\pm 0.21)$, Al 28.31 $(28.35 \pm 0.09), \quad \mathrm{Au} 47.35 \quad(47.49 \pm 0.21), \quad \mathrm{Cu} 33.16$ $(33.15 \pm 0.08)$, Pt $41.45(41.63 \pm 0.21)$, Ta 41.50 (41.51 $\pm 0.17$ ), MgO 26.96 (26.94 $\pm 0.17$ ), and for diamond $2.366(2.38 \pm 0.01) \mathrm{J} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$. Therefore the constructed EOSs can also be used for a compact representation of thermodynamic functions of substances without phase transitions from $10-15 \mathrm{~K}$ up to the melting temperature at room pressure.

Figure 8 shows the deviations of the calculated pressure on room-temperature isotherms from selected experimental (without recalibration) and theoretical data. The deviations of
data of Dewaele et al. ${ }^{24,26}$ for $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}$, and W recalibrated using the scale (16) are very small, like those shown in Fig. 4. In the work of Chijioke et al., ${ }^{43}$ shock-wave data were used for calculation of room-temperature isotherms of $\mathrm{Al}, \mathrm{Cu}, \mathrm{Ta}$, and W . They used ultrasonic measurements of bulk moduli at low pressure and considered the effect of stress in the shock-wave data, however, their roomtemperature isotherms do not always coincide with ours. At the same $x$, the pressure on room-temperature isotherms ${ }^{43}$ is higher than found here for Al and Pt , lower for $\mathrm{Au}, \mathrm{Cu}$, and W [Fig. 6(e)] and almost coincides for Ta. A similar situation occurs with the room-temperature isotherms reduced from shock-wave data in Refs. 29 and 56, except Al, for which data of Ref. 56 are in excellent agreement with our results.

The room-temperature isotherms reduced from the shockwave data of Carter et al. ${ }^{10}$ agree with ours up to 100 GPa for Cu and Ag , but above 100 GPa the pressure from Ref. 10 is higher. Figure 8 also shows unpublished data of Dewaele et al. ${ }^{86}$ for Ag , measured at the same conditions as in Ref. 24. Deviations of these data are shown in two variants: first, calibrating the pressure with the scale, ${ }^{24}$ second, using our scale (16). These measurements confirm, once again, the most recent calibrations of the ruby pressure scale.

For Ta the room-temperature isotherms ${ }^{29,43}$ reduced from shock-wave data agree well with our isotherms. In the measurements of Cynn and Yoo, ${ }^{84}$ carried out in the argon medium, the pressure was determined using the gold pressure scale with parameters $K_{0}=166.6 \mathrm{GPa}$ and $K^{\prime}=5.5$. In Fig. 8 these data are shown after recalculation using the EOS of Au


FIG. 6. Comparison of thermodynamic functions of tungsten calculated from our EOS (parameters from Table I) with experimental and theoretical data. (a) $C_{p}$ and $C_{V}$, experimental data from Refs. 46-48. (b) Relative deviations of the calculated $C_{p}$ from experimental measurements (Refs. 47 and 49). (c) Thermal expansion coefficient in comparison with Refs. 50-52. (d) Bulk moduli in comparison with Refs. 53-55; dotted lines correspond to the variation of $K^{\prime}$ in the range $\pm 0.2$. (e) Deviations of the calculated pressure from experimental and theoretical data (Refs. 24, 56, 28, and 43) (f) Calculated 300 and 3000 K isotherms and shock adiabats in comparison with experiments (Refs. 24, 30, and 28). (g) Grüneisen parameters. (h) Calculated shock-wave velocity versus particle velocity in comparison with the Shock Wave Database (Ref. 30).
(Table I); one can see that the recalculated isotherm ${ }^{84}$ agrees with ours. In the measurements of Hanfland et al., ${ }^{83}$ carried out in the neon and Na pressure medium, pressure was calculated using the ruby scale of Mao et al. ${ }^{6}$ Figure 8 shows the deviation of the isotherms ${ }^{83}$ calculated using two set of parameters: $K_{0}=207.6 \mathrm{GPa}, K^{\prime}=2.85$ and $K_{0}=199 \mathrm{GPa}, K^{\prime}$ $=3.95$.

We have slightly modified our previous version ${ }^{45,98}$ of the equations of state of gold and MgO , taking into account the measurements of Refs. 89 and 90 in He pressure-transmitting medium and measurements of Ref. 43 in hydrogen pressuretransmitting medium. Pressures in Fig. 8 are calculated for these data using the ruby scale (16). It turned out that cell parameters calculated from (111) and (200) x-ray reflections ${ }^{89}$ are significantly different, which contributes to the uncertainty in pressure calibration. The adopted value
$K^{\prime}=6.0$ for gold (Table I) is consistent with measurements from Refs. 24, 43, and 90 and with pressure calculated from (111) reflections. ${ }^{89}$ For Pt we see a satisfactory agreement between the isotherms in Fig. 8, except the isotherm of Ref. 99 , where the recommended $K^{\prime}=4.8 \pm 0.3$ is much lower than our value $K^{\prime}=5.3$.

The deviations of the room-temperature isotherms of MgO from the measurements of Speziale et al. ${ }^{93}$ are shown in Fig. 8 in two variants: the original data, ${ }^{93}$ where the pressure was determined using the Mao et al. ${ }^{6}$ scale, and recalibrated using Eq. (16). The ideal consistency of the recalibrated isotherm is achieved with $K^{\prime}=4.4$, but this leads to inconsistencies with other measurements, ${ }^{19,100}$ which will be considered elsewhere. ${ }^{96}$ Figure 8 also shows the differences between the measurements of Occelli et al. ${ }^{25}$ for diamond recalibrated using Eq. (16), and the room-temperature iso-


FIG. 7. Comparison of the calculated bulk moduli of $\mathrm{Al}, \mathrm{Cu}, \mathrm{Ta}$, $\mathrm{Ag}, \mathrm{Au}, \mathrm{Pt}$, diamond, and MgO with ultrasonic measurements [for tungsten see Fig. 6(d)]. Dotted lines correspond to the variation of $K^{\prime}$ in the range $\pm 0.2$, except platinum for which the range $\pm 0.5$ is shown; the top dotted line corresponds to the lower value of $K^{\prime}$. Experimental data are from Refs. 57-62 for Al, Refs. 63-66 for Cu, Refs. 55 and $67-70$ for Ta, Refs. 71, 65, and 66 for Ag, Refs. 71, 65, 72, 73, and 66 for Au, Ref. 74 for Pt, Refs. 75-77 for MgO, and Refs. 78 and 15 for diamond.
therm of diamond with parameters $K_{0}=443.2 \mathrm{GPa}$ and $K^{\prime}$ $=3.77$ (Table I).

Summing up, for a number of reference materials we have developed a unified thermodynamic model capable of describing a very diverse set of experimental data (thermochemistry, $P-V-T$ EOSs, ultrasonic measurements) within the experimental error bars. The room-temperature isotherms of reference metals allowed us to recalibrate the ruby pressure scale. The proposed scale in the form (16) agrees very well with the recalibrated measurements of Dewaele et al., ${ }^{24}$ and is consistent with independent quasihydrostatic measurements, but only when these measurements are recalibrated.

## IV. DISCUSSION AND SUMMARY

Let us compare recent ruby pressure scales extrapolated to 300 GPa and discuss the importance of the chosen functional form of $P(\lambda)$ (Fig. 9). At given $\lambda$ we have calculated pressure (and its deviations from our scale). The scatter of the calculated pressures exceeds $\pm 30 \mathrm{GPa}$ at a pressure of 300 GPa , but is much smaller for ruby scales published after 2003.

At the pressure of 150 GPa all recent scales described by various functional dependences are consistent within $\pm 3-4 \mathrm{GPa}$, but at higher pressures they begin to diverge. In


FIG. 8. Differences between our calculated room-temperature isotherms (Table I) and previous determinations: $\Delta P=P_{\text {old }}-P_{\text {new. }}$. DO ruby" means calibration done with the present ruby scale. Experimental and theoretical data are from Refs. 79, 56, 80, 24, 43, and 81 for Al , Refs. 56, 24, 29, 43, 10, 6, and 82 for Cu, Refs. 43, 83, 84, 24, 26, and 82 for Ta, Refs. 10, 85, 6, and 86 for Ag (see text), Refs. 87, 24, 88, 43, 89, and 90 for Au , Refs. 24, 91, 92, 87, 29, and 43 for Pt, Refs. 93 and 94 for MgO, and Refs. 95 and 25 for diamond.
our view, this is largely related to the different forms of $P(\lambda)$, which is easily seen in the example of our scale in the form Eq. (1) with parameters $A=1885 \mathrm{GPa}$ and $B=11$ (Fig. 3) which as we mentioned, is good up to 85 GPa , and above 85 GPa has parameters $A=1975 \mathrm{GPa}$ and $B=8.59$. However,
instead of these two scales it is sufficient to use a single scale described by Eq. (16) and recommended here.

The scale of Chijioke et al. ${ }^{43}$ in the form Eq. (1) at pressures above 120 GPa considerably deviates from our scale (16). However, they also represented their scale by Eq. (4)


FIG. 9. Difference between published ruby scales extrapolated to pressure 300 GPa at given value $\lambda: \Delta P=P_{\text {Eq. (16) }}-P_{\text {other. }}$. MBX 1986-Mao et al. (Ref. 6) AGZS 1987 2—Aleksandrov et al. (Ref. 12) calibration using Eq. (2) with parameters $A=1892 \pm 13 \mathrm{GPa}$ and $m=6.4$; H 2003—Holzapfel (Ref. 20), DO 2003 1—Dorogokupets and Oganov (Ref. 21) calibration using Eq. (1) with parameters $A$ $=1871 \mathrm{GPa}$ and $B=10.06$; KLS 2004—Kunc et al. (Refs. 22, 23, and 41) calibration in the form $P=1860 \times(\Delta \lambda / \lambda)[1+7.75(\Delta \lambda / \lambda)]$; DLM $2004 —$ Dewaele et al. (Ref. 24) calibration using Eq. (1) with parameters $A=1904 \mathrm{GPa}$ and $B=9.5$; CNSS 2005 1-Chijioke et al. (Ref. 43) calibration using Eq. (1) with parameters $A$ $=1873 \pm 6.7$ GPa and $B=10.82 \pm 0.14$; H 2005—Holzapfel (Ref. 44) CNSS 2005 4-Chijioke et al. (Ref. 43) calibration in the form $P$ $=1794 \times(\Delta \lambda / \lambda)[1+8.68(\Delta \lambda / \lambda)]$. Dashed lines correspond to $\pm 2 \%$ deviations.
with parameters $A=1794 \pm 8.4 \mathrm{GPa}$ and $\mu=8.68 \pm 0.15$; this function is consistent with our scale (16) also above 120 GPa and does not contradict recommendations of other authors.

The ruby pressure scale in the form (4) is discussed in Refs. 20, 22, 23, and 41. These authors remark that the twoparametrical scale (4) with $A=1820 \mathrm{GPa}$ and $\mu=7.9$ practically coincides with the three-parametrical scale of Holzapfel ${ }^{20}$ up to a pressure of 200 GPa . At the same time Kunc et al. ${ }^{22,23,41}$ recommended a ruby pressure scale in the form (4) with parameters $A=1860 \mathrm{GPa}$ and $\mu=7.75$, which agrees with our scale to within $1 \%$ (Fig. 9).

Holzapfel ${ }^{44}$ has re-analyzed the data of Dewaele et al. ${ }^{24}$ and Occelli et al. ${ }^{25}$ and has obtained a ruby pressure scale with parameters $A=1845 \mathrm{GPa}, B=14.7, C=7.5$ for Eq. (3), which rather considerably differs from his previous ruby pressure scale, ${ }^{20}$ but deviations from our scale only slightly exceed $2 \%$ in the range $100-220 \mathrm{GPa}$.

From the comparison of the proposed ruby pressure scale (16) and the scale of Chijioke et al. ${ }^{43}$ in the form (4) it is possible to draw conclusions about the most appropriate functional form for $P(\lambda)$. If one considers these scales as closest to the truth, then the scale (16) in the form (2) has doubtless advantage of having the parameter $A=1884 \mathrm{GPa}$ close to the direct measurement $A=1875 \pm 30 \mathrm{GPa}$ in Ref. 39 . The scale of Chijioke et al. ${ }^{43}$ in the form (4) has a considerably smaller value of the parameter $A=1794 \pm 8.4 \mathrm{GPa}$, which is very different from direct measurements at low pressure. ${ }^{39}$



FIG. 10. Comparison of MgO and Au pressure scale (Tables IV and IX) with other pressure scales. (a) compares our Au and MgO pressure scales using measurements from Ref. 105. (b) is based on measurements from Ref. 106 (done simultaneously for MgO and Au ) and calibrated using our Au and MgO EOSs (DO) or using the Au pressure scales from Ref. 73 (Shim et al. Au EOS), Ref. 107 (Tsuchiya Au EOS), Ref. 92 (Fei et al. Au EOS), and MgO pressure scale from Ref. 93 (Speziale et al. MgO EOS).

A comparison of the room-temperature isotherms of Au and $\mathrm{Pt}, \mathrm{Au}$ and Ag has been done by us earlier ${ }^{45}$ on the basis of independent determinations of Akahama et al., ${ }^{101}$ and has shown quite a satisfactory agreement despite the fact that measurements ${ }^{101}$ have been carried out in nonhydrostatic conditions. The self-consistency of our EOSs can be checked not only on room-temperature isotherms, but also at higher temperatures and pressures, using simultaneous PVT measurements of the unit cell parameters of $\mathrm{MgO}, \mathrm{Pt}$, and Au by Matsui and Nishiyama, ${ }^{102}$ Nishiyama et al., ${ }^{103}$ and Fei et al. ${ }^{92,104}$ Using the measured cell parameters of $\mathrm{MgO}, \mathrm{Pt}$, and Au we have calculated pressures for these materials at different temperatures. One can see (Fig. 9 in Ref. 45) excellent agreement with the data of Fei et al., ${ }^{92,104}$ whereas some systematic difference is present for the measurements from Refs. 102 and 103. Very recent studies of Inoue et al. ${ }^{105}$ and Hirose et al. ${ }^{106}$ give us a possibility to compare the EOSs of Au and MgO at very high $P-T$ parameters. Figure 10 (a) compares the pressure scales of MgO and Au , calculated from the measurements of Inoue et al. ${ }^{105}$ and checked against our Au and MgO EOSs at temperatures from 300 to 1773 K and pressures from 13 up to 22 GPa . Figure 10 (b) compares the pressure scales of MgO and Au at pressures above 100 GPa based on the measurements of Hirose et al., ${ }^{106}$ where EOSs


FIG. 11. The difference $\Delta P=P_{\mathrm{DO}}-P_{\text {other }}$ between calculated pressure (Table IV) on 1000, 2000, and 3000 K isotherms of Au and pressures from Jamieson et al. (Ref. 87), Anderson et al. (Ref. 108), Shim et al. (Ref. 73), Tsuchiya (Ref. 107), Greeff and Graf (Ref. 110), and Souvatzis et al. (Ref. 109). EOSs of gold, plotted against normalized unit-cell volume (see pressure at given $x=V / V_{0}$ in Table IV).
of MgO (Ref. 93) and Au (Ref. 107) have been compared. While Au EOS (Ref. 107) gives pressures up to 5 GPa lower than MgO EOS (Ref. 93) at 2330 K , our EOSs of MgO and Au differ by less than 3 GPa . Comparing MgO EOS (Ref. 93) and Au EOSs from Refs. 73 and 92 one can see that Au EOS from Refs. 73 and 92 underestimate pressure at 2330 K by $\sim 10 \mathrm{GPa}$ in comparison with MgO EOS. ${ }^{93}$

Finally, let us comment on the EOS of Au, which has been recommended as a primary pressure standard at elevated temperatures, ${ }^{87}$ but for which very different EOSs have been suggested in subsequent works (Refs. 108 and 73). The differences between our data and previously published room-temperature isotherms of Au in coordinates $\Delta P-P$ is shown in Fig. 8, and Fig. 11 shows the differences on 1000, 2000 , and 3000 K isotherms in coordinates $\Delta P-x$. While the differences at room temperature are entirely due to the difference in $K_{0}$ and $K^{\prime}$, at elevated temperatures errors in the thermal pressure must also be considered. In Ref. 108, thermal pressure was evaluated using empirical relations leading to large differences from our results. We find good agreement with the results of first-principles calculations of Refs. 107
and 109 especially at high temperatures, but there are differences from calculations of Ref. 110, where for the roomtemperature isotherm $K^{\prime}=5.5$ was adopted. We suggest that the EOS of gold reported in Table IV gives a reliable pressure standard.

Another popular pressure standard, platinum, is less preferable because at high pressures and temperatures it was found to react with diamond forming PtC (Ref. 111) and because of the uncertainty in the temperature dependence of the adiabatic bulk modulus (see Ref. 74 and discussion in the text).

Tables II-X with the calculated isotherms of $\mathrm{Ag}, \mathrm{Al}, \mathrm{Cu}$, $\mathrm{Pt}, \mathrm{Ta}, \mathrm{W}, \mathrm{MgO}$, and diamond extending to $\sim 200 \mathrm{GPa}$ and 3000 K are given in the Appendix. ${ }^{112}$ We recommend these tables for pressure calibration in this $P-T$ range.

## V. CONCLUDING REMARKS

We have obtained internally consistent semiempirical EOSs of $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}, \mathrm{W}$, and MgO in a wide pressuretemperature range on the basis of a simultaneous analysis of thermochemical, x-ray, ultrasonic, and shock-wave data. Our EOSs are consistent with all recent experimental data. The EOSs of $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}$ and W , combined with the precise measurements of Dewaele et al., ${ }^{24,26}$ allowed us to obtain a new calibration of the ruby pressure scale. The cross check with the independent data of Matsui and Nishiyama, ${ }^{102}$ Nishiyama et al., ${ }^{103}$ Fei et al., ${ }^{92,104}$ Inoue et al., ${ }^{105}$ and Hirose et al. ${ }^{106}$ confirms the obtained EOSs of $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}, \mathrm{W}$, and MgO .

The revised ruby pressure scale has a simple functional form capable of representing the $P(\lambda)$ dependence in the whole pressure range $0-300 \mathrm{GPa}$ by a single equation. The obtained ruby pressure scale agrees to within $2 \%$ with other recent ruby pressure scales (Refs. 20-24, 41, 43, and 44), but has a number of advantages. First, it is consistent with roomtemperature isotherms of $\mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}$, and W obtained here on the basis of our unified thermodynamic formalism. Second, it does not contradict recent EOSs of diamond. ${ }^{22,23,41,113}$ Third, and most important, the obtained $P-V-T$ EOSs enable consistent pressure calibration using either the ruby scale or EOSs of any of the reference substances studied here ( $\mathrm{Ag}, \mathrm{Al}, \mathrm{Au}, \mathrm{Cu}, \mathrm{Pt}, \mathrm{Ta}, \mathrm{W}$, diamond, MgO ). This solves problems of inconsistency between different pressure scales and enables accurate pressure calibration at elevated temperatures, where the ruby scale cannot be used.

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## APPENDIX

TABLE II. Isochors for Ag. Pressure (in GPa) as a function of compression $\left(x=V / V_{0}\right)$ and temperature (K).

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 2500 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 4.013 | 9.791 | 12.786 |
| 0.95 | 5.972 | 9.999 | 15.797 | 18.771 |
| 0.9 | 14.419 | 18.475 | 24.325 | 27.297 |
| 0.85 | 26.319 | 30.421 | 36.353 | 39.350 |
| 0.8 | 43.076 | 47.244 | 53.294 | 56.340 |
| 0.75 | 66.745 | 70.998 | 77.206 | 80.328 |
| 0.7 | 100.371 | 104.735 | 111.146 | 114.371 |
| 0.65 | 148.568 | 153.067 | 159.735 | 163.093 |
| 0.6 | 218.470 | 223.132 | 230.123 | 233.648 |

TABLE III. Isochors for Al.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 2500 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 3.764 | 10.168 | 13.913 |
| 0.95 | 4.193 | 7.747 | 13.739 | 17.301 |
| 0.9 | 9.736 | 20.310 | 25.629 | 22.121 |
| 0.85 | 17.061 | 29.917 | 35.000 | 28.817 |
| 0.8 | 26.767 | 42.771 | 47.699 | 38.004 |
| 0.75 | 39.684 | 60.046 | 64.905 | 50.537 |
| 0.7 | 56.986 | 83.432 | 88.314 | 67.613 |
| 0.65 | 80.366 | 115.417 | 120.410 | 90.947 |
| 0.6 | 112.309 | 159.749 | 164.941 | 123.039 |
| 0.55 | 156.562 | 222.232 | 227.716 | 167.640 |
| 0.5 | 218.931 |  | 230.556 |  |

TABLE IV. Isochors for Au.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 2500 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.00 | 4.96 | 12.11 | 15.96 |
| 0.95 | 9.96 | 14.72 | 21.55 | 25.20 |
| 0.9 | 23.98 | 28.57 | 35.12 | 38.58 |
| 0.85 | 43.65 | 48.08 | 54.42 | 57.71 |
| 0.8 | 71.21 | 75.53 | 81.71 | 84.87 |
| 0.75 | 109.98 | 114.21 | 120.29 | 123.36 |
| 0.7 | 164.83 | 169.00 | 175.03 | 178.06 |
| 0.65 | 243.11 | 247.25 | 253.31 | 256.35 |

TABLE V. Isochors for Cu .

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 2500 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 4.789 | 12.152 | 16.534 |
| 0.95 | 7.845 | 12.644 | 19.909 | 24.143 |
| 0.9 | 18.575 | 23.424 | 30.683 | 34.791 |
| 0.85 | 33.222 | 38.161 | 45.510 | 49.536 |
| 0.8 | 53.237 | 58.308 | 65.844 | 69.848 |
| 0.75 | 80.689 | 85.932 | 93.753 | 97.810 |
| 0.7 | 118.576 | 124.032 | 132.235 | 136.428 |
| 0.65 | 171.322 | 177.032 | 185.717 | 190.127 |
| 0.6 | 245.599 | 251.600 | 260.876 | 265.581 |

TABLE VI. Isochors for Pt.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 3000 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 5.309 | 12.864 | 20.349 |
| 0.95 | 16.207 | 21.219 | 28.485 | 35.822 |
| 0.9 | 38.307 | 43.111 | 50.199 | 57.476 |
| 0.85 | 68.389 | 73.069 | 80.079 | 87.376 |
| 0.8 | 109.388 | 114.016 | 121.043 | 128.434 |
| 0.75 | 165.473 | 170.119 | 177.253 | 184.808 |
| 0.7 | 242.676 | 247.403 | 254.730 | 262.523 |

TABLE VII. Isochors for Ta.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 3000 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 2.709 | 6.538 | 10.351 |
| 0.95 | 10.818 | 13.465 | 17.274 | 21.111 |
| 0.9 | 24.582 | 27.219 | 31.074 | 35.009 |
| 0.85 | 42.131 | 44.803 | 48.762 | 52.855 |
| 0.8 | 64.588 | 67.334 | 71.451 | 75.749 |
| 0.75 | 93.474 | 96.332 | 100.654 | 105.203 |
| 0.7 | 130.887 | 133.891 | 138.468 | 143.309 |
| 0.65 | 179.768 | 182.953 | 187.836 | 193.013 |
| 0.6 | 244.325 | 247.727 | 252.971 | 258.537 |

TABLE VIII. Isochors for W.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 3000 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 2.934 | 7.461 | 12.670 |
| 0.95 | 17.456 | 20.238 | 24.520 | 29.361 |
| 0.9 | 40.047 | 42.703 | 46.783 | 51.310 |
| 0.85 | 69.323 | 71.877 | 75.797 | 80.072 |
| 0.8 | 107.377 | 109.855 | 113.658 | 117.743 |
| 0.75 | 157.082 | 159.512 | 163.239 | 167.200 |
| 0.7 | 222.444 | 224.854 | 228.550 | 232.448 |

TABLE IX. Isochors for MgO.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 3000 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 4.174 | 10.354 | 16.286 |
| 0.95 | 9.147 | 13.263 | 19.478 | 25.516 |
| 0.9 | 20.988 | 25.076 | 31.374 | 37.560 |
| 0.85 | 36.338 | 40.425 | 46.857 | 53.236 |
| 0.8 | 56.295 | 60.409 | 67.029 | 73.651 |
| 0.75 | 82.371 | 86.534 | 93.401 | 100.325 |
| 0.7 | 116.670 | 120.904 | 128.081 | 135.372 |
| 0.65 | 162.179 | 166.501 | 174.059 | 181.794 |
| 0.6 | 223.226 | 227.647 | 243.665 | 243.935 |

TABLE X. Isochors for diamond.

| $x=V / V_{0}$ | 298.15 | 1000 | 2000 | 3000 |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 2.706 | 8.466 | 14.788 |
| 0.95 | 25.028 | 27.518 | 32.942 | 38.930 |
| 0.9 | 56.828 | 59.201 | 64.491 | 70.359 |
| 0.85 | 97.318 | 99.641 | 104.940 | 110.843 |
| 0.8 | 149.061 | 151.377 | 156.790 | 162.847 |
| 0.75 | 215.532 | 217.870 | 223.477 | 229.779 |
| 0.7 | 301.514 | 303.890 | 309.757 | 316.383 |

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${ }^{1}$ C. E. Weir, E. R. Lippincott, A. Van Valkenburg, and E. N. Bunting, J. Res. Natl. Bur. Stand., Sect. A 63A, 55 (1959); J. C. Jamieson, A. W. Lawson, and N. D. Nachtrieb, Rev. Sci. Instrum. 30, 1016 (1959).
${ }^{2}$ R. A. Forman, G. J. Piermarini, J. D. Barnett, and S. Block, Science 176, 284 (1972).
${ }^{3}$ G. J. Piermarini, S. Block, J. D. Barnett, and R. A. Forman, J. Appl. Phys. 46, 2774 (1975).
${ }^{4}$ G. J. Piermarini and S. Block, Rev. Sci. Instrum. 46, 973 (1975).
${ }^{5}$ H. K. Mao, P. M. Bell, J. W. Shaner, and D. J. Steinberg, J. Appl. Phys. 49, 3276 (1978).
${ }^{6}$ H. K. Mao, J. P. Xu, and P. Bell, J. Geophys. Res. 91, 4673 (1986).
${ }^{7}$ A. Jayaraman, Rev. Mod. Phys. 55, 65 (1983).
${ }^{8}$ A. Jayaraman, Rev. Sci. Instrum. 57, 1013 (1986).
${ }^{9}$ G. J. Piermarini, J. Res. Natl. Inst. Stand. Technol. 106, 889 (2001).
${ }^{10}$ W. J. Carter, S. P. Marsh, J. N. Fritz, and R. G. McQueen, in Accurate Characterization of the High Pressure Environment, edited by E. C. Lloyd (National Bureau of Standards, Washington, DC, 1971), NBS Spec. Publ., Vol. 326, p. 147.
${ }^{11}$ D. L. Decker, J. Appl. Phys. 42, 3239 (1971).
${ }^{12}$ I. V. Aleksandrov, A. F. Goncharov, A. N. Zisman, and S. M. Stishov, Zh. Eksp. Teor. Fiz. 93, 680 (1987) [Sov. Phys. JETP 66, 384 (1987)].
${ }^{13}$ O. H. Nielsen, Phys. Rev. B 34, 5808 (1986).
${ }^{14}$ M. Hanfland, K. Syassen, S. Fahy, S. G. Louie, and M. L. Cohen, Phys. Rev. B 31, 6896 (1985).
${ }^{15}$ H. J. McSkimin and P. Andreatch, J. Appl. Phys. 43, 2944 (1972).
${ }^{16}$ R. J. Hemley, C. S. Zha, A. P. Jephcoat, H. K. Mao, L. W. Finger, and D. E. Cox, Phys. Rev. B 39, 11820 (1989).
${ }^{17}$ R. G. McQueen, S. P. Marsh, J. W. Taylor, J. N. Fritz, and W. J. Carter, in High-Velocity Impact Phenomena, edited by R. Kinslow (Academic, New York, 1970), p. 293.
${ }^{18}$ P. M. Bell and H. K. Mao, Carnegie Inst. Washington Publ. 80, 404 (1981).
${ }^{19}$ C. S. Zha, H. K. Mao, and R. J. Hemley, Proc. Natl. Acad. Sci. U.S.A. 97, 13494 (2000).
${ }^{20}$ W. B. Holzapfel, J. Appl. Phys. 93, 1813 (2003).
${ }^{21}$ P. I. Dorogokupets and A. R Oganov, Dokl. Earth Sci. 391A, 854 (2003).
${ }^{22}$ K. Kunc, I. Loa, and K. Syassen, Phys. Rev. B 68, 094107 (2003).
${ }^{23}$ K. Kunc, I. Loa, and K. Syassen, High Press. Res. 24, 101 (2004).
${ }^{24}$ A. Dewaele, P. Loubeyre, and M. Mezouar, Phys. Rev. B 70, 094112 (2004).
${ }^{25}$ F. Occelli, P. Loubeyre, and R. Letoullec, Nat. Mater. 2, 151 (2003).
${ }^{26}$ A. Dewaele, P. Loubeyre, and M. Mezouar, Phys. Rev. B 69, 092106 (2004).
${ }^{27}$ W. J. Nellis, J. A. Moriarty, A. C. Mitchell, M. Ross, R. G. Dandrea, N. W. Ashcroft, N. C. Holmes, and G. R. Gather, Phys. Rev. Lett. 60, 1414 (1988).
${ }^{28}$ R. S. Hixson and J. N. Fritz, J. Appl. Phys. 71, 1721 (1992).
${ }^{29}$ Y. Wang, R. Ahuja, and B. Johansson, J. Appl. Phys. 92, 6616 (2002).
${ }^{30}$ K. V. Khishchenko, P. R. Levashov and I. V. Lomonosov, Shock

Wave Database. Available from http://teos.ficp.ac.ru/ rusbank(2001).
${ }^{31}$ V. N. Zharkov and V. A. Kalinin, Equations of State of Solids at High Pressures and Temperatures (Consultants Bureau, New York, 1971).
${ }^{32}$ P. Vinet, J. Ferrante, J. Rose, and J. Smith, J. Geophys. Res. 92, 9319 (1987).
${ }^{33}$ A. M. Kut'in and D. V. Pyadushkin, Russ. J. Phys. Chem. 72, 1567 (1998).
${ }^{34}$ A. M. Kut'in, D. V. Pyadushkin, and E. A. Bykova, Russ. J. Phys. Chem. 72, 1573 (1998).
${ }^{35}$ L. V. Al'tshuler, S. E. Brusnikin, and E. A. Kuz'menkov, J. Appl. Mech. Tech. Phys. 28, 129 (1987).
${ }^{36}$ A. R. Oganov and P. I. Dorogokupets, J. Phys.: Condens. Matter 16, 1351 (2004).
${ }^{37}$ M. I. Katsnelson, A. F. Maksyutov, and A. V. Trefilov, Phys. Met. Metallogr. 95, 304 (2003).
${ }^{38}$ K. Wang and R. R. Reeber, Mater. Sci. Eng., R. R23, 101 (1998).
${ }^{39}$ O. Grasset, High Press. Res. 21, 139 (2001).
${ }^{40}$ K. Nakano, Y. Akahama, Y. Ohishi, and H. Kawamura, Jpn. J. Appl. Phys. 39, 1249 (2000).
${ }^{41}$ K. Syassen, unpublished notes (2005).
${ }^{42}$ N. Funamori and R. Jeanloz, Science 278, 1109 (1997); T. Mashimo, K. Tsumoto, K. Nakamura, Y. Noguchi, K. Fukuoka, and Y. Syono, Geophys. Res. Lett. 27, 2021 (2000); J. F. Lin, O. Degtyareva, C. T. Prewitt, P. Dera, N. Sata, E. Gregoryanz, H. K. Mao, and R. J. Hemley, Nat. Mater. 3, 389 (2004); A. R. Oganov and S. Ono, Proc. Natl. Acad. Sci. U.S.A. 102, 10828 (2005); R. Caracas and R. E. Cohen, Geophys. Res. Lett. 32, L06303 (2005); J. Tsuchiya, T. Tsuchiya, and R. M. Wentzcovitch, Phys. Rev. B 72, 020103 (2005).
${ }^{43}$ A. D. Chijioke, W. J. Nellis, A. Soldatov, and I. F. Silvera, J. Appl. Phys. 98, 073526 (2005); 98, 114905 (2005).
${ }^{44}$ W. B. Holzapfel, High Press. Res. 25, 187 (2005).
${ }^{45}$ P. I. Dorogokupets and A. R. Oganov, in Proceedings Joint 20th AIRAPT \& 43th EHPRG International Conference on High Pressure Science and Technology, 27 June-1 July 2005, Karlsruhe Germany (Forschungszentrum Karlsruhe, Karlsruhe, 2005).
${ }^{46}$ L. V. Gurvich, I. V. Veiz, V. A. Medvedev et al., Thermodynamic Properties of Individual Substances (Nauka, Moscow, 1981), Vol. 4, Book 2.
${ }^{47}$ K. White and S. Collocott, J. Phys. Chem. Ref. Data 13, 1251 (1984).
${ }^{48}$ M. W. Chase, Jr., NIST-JANAF Thermochemical Tables. 4th Ed., J. Phys. Chem. Ref. Data Monogr. No. 9 (1998).
${ }^{49}$ Y. S. Touloukian and E. H. Buiko, in Thermophysical Properties of Matter (Plenum Press, New York, 1970), Vol. 4.
${ }^{50}$ S. I. Novikova, Thermal Expansion of Solids (Nauka, Moscow, 1974), in Russian.
${ }^{51}$ Y. S. Touloukian, R. K. Kirby, R. E. Taylor, and P. D. Desai, in Thermophysical Properties of Matter (Plenum, New York, 1975), Vol. 12.
${ }^{52}$ L. S. Dubrovinsky and S. K. Saxena, Phys. Chem. Miner. 24, 547 (1997).
${ }^{53}$ B. T. Bernstein, J. Appl. Phys. 33, 2140 (1962).
${ }^{54}$ R. Lowrie and A. M. Gonas, J. Appl. Phys. 36, 2189 (1965).
${ }^{55}$ F. H. Featherston and J. R. Neighbours, Phys. Rev. 130, 1324 (1963).
${ }^{56}$ Y. Wang, D. Chen, and X. Zhang, Phys. Rev. Lett. 84, 3220
(2000).
${ }^{57}$ J. L. Tallon and A. Wolfenden, J. Phys. Chem. Solids 40, 831 (1979).
${ }^{58}$ G. N. Kamm and G. A. Alers, J. Appl. Phys. 35, 327 (1964).
${ }^{59}$ P. S. Ho and A. L. Ruoff, J. Appl. Phys. 40, 3151 (1969).
${ }^{60}$ D. Gerlich and E. S. Fisher, J. Phys. Chem. Solids 30, 1197 (1969).
${ }^{61}$ R. B. McLellan and T. Ishikawan, J. Phys. Chem. Solids 48, 603 (1987).
${ }^{62}$ K. Wang and R. R. Reeber, Philos. Mag. A 80, 1629 (2000).
${ }^{63}$ W. C. Overton and J. Gaffney, Phys. Rev. 98, 969 (1955).
${ }^{64}$ Y. A. Chang and R. Hultgren, J. Phys. Chem. 69, 4162 (1965).
${ }^{65}$ Y. A. Chang and L. Himmel, J. Appl. Phys. 37, 3567 (1966).
${ }^{66}$ W. Holzapfel, M. Hartwig, and W. Sievers, J. Phys. Chem. Ref. Data 30, 515 (2001).
${ }^{67}$ R. G. Leisure, D. K. Hsu, and B. A. Seiber, J. Appl. Phys. 44, 3394 (1973).
${ }^{68}$ D. I. Bolef, J. Appl. Phys. 32, 100 (1961).
${ }^{69}$ N. Soga, J. Appl. Phys. 37, 3416 (1966).
${ }^{70}$ O. Gulseren and R. E. Cohen, Phys. Rev. B 65, 064103 (2002).
${ }^{71}$ J. R. Neighbours and G. A. Alers, Phys. Rev. 111, 707 (1958).
${ }^{72}$ S. M. Collard and R. B. McLellan, Acta Metall. Mater. 39, 3143 (1991).
${ }^{73}$ S. H. Shim, T. S. Duffy, and K. Takemura, Earth Planet. Sci. Lett. 203, 729 (2002).
${ }^{74}$ S. M. Collard and R. B. McLellan, Acta Metall. Mater. 40, 699 (1992).
${ }^{75}$ D. G. Isaak, O. L. Anderson, and T. Goto, Phys. Chem. Miner. 16, 704 (1989).
${ }^{76}$ Y. Sumino, O. L. Anderson, and I. Suzuki, Phys. Chem. Miner. 9, 38 (1983).
${ }^{77}$ O. L. Anderson and P. Andreatch, J. Am. Ceram. Soc. 49, 404 (1966).
${ }^{78}$ E. S. Zouboulis, M. Grimsditch, A. K. Ramdas, and S. Rodriguez, Phys. Rev. B 57, 2889 (1998).
${ }^{79}$ W. J. Nellis, J. A. Moriarty, A. C. Mitchell, M. Ross, R. G. Dandrea, N. W. Ashcroft, N. C. Holmes, and G. R. Gather, Phys. Rev. Lett. 60, 1414 (1988).
${ }^{80}$ R. G. Greene, H. Luo, and A. L. Ruoff, Phys. Rev. Lett. 73, 2075 (1994).
${ }^{81}$ N. Nishimura, K. Kinoshita, Y. Akahama, and H. Kawamura, in Proceedings Joint 20th AIRAPT \& 43th EHPRG International Conference on High Pressure Science and Technology, 27 June-1 July 2005, Karlsruhe Germany (Forschungszentrum Karlsruhe, Karlsruhe, 2005).
${ }^{82}$ J. Hu, H. K. Mao, and P. M. Bell, High Temp. - High Press. 16, 495 (1984).
${ }^{83}$ M. Hanfland, K. Syassen, and J. Köhler, J. Appl. Phys. 91, 4143 (2002).
${ }^{84}$ H. Cynn and C. S. Yoo, Phys. Rev. B 59, 8526 (1999).
${ }^{85}$ K. Syassen and W. B. Holzapfel, J. Appl. Phys. 49, 4427 (1978).
${ }^{86}$ A. Dewaele, P. Loubeyre, and M. Mezouar, unpublished data for Ag.
${ }^{87}$ J. C. Jamieson, J. N. Fritz, and M. H. Manghnani, in High-

Pressure Research in Geophysics, edited by S. Akimoto and M. H. Manghnani (Center for Academic Publications, Tokyo, 1982), p. 27.
${ }^{88}$ D. L. Heinz and R. Jeanloz, J. Appl. Phys. 55, 885 (1984).
${ }^{89}$ K. Takemura, J. Appl. Phys. 89, 662 (2001).
${ }^{90}$ T. Yagi, K. Okabe, N. Nishiyama, A. Kubo, and T. Kikegawa, Phys. Earth Planet. Inter. 143-144, 81 (2004).
${ }^{91}$ N. C. Holmes, J. A. Moriarty, G. R. Gather, and W. J. Nellis, J. Appl. Phys. 66, 2962 (1989).
${ }^{92}$ Y. Fei, J. Li, K. Hirose, W. Minarik, J. Van Orman, C. Sanloup, W. Van Westrenen, T. Komabayashi, and K. Funakoshi, Phys. Earth Planet. Inter. 143-144, 515 (2004).
${ }^{93}$ S. Speziale, C. C. Zha, T. S. Duffy, R. J. Hemley, and H. K. Mao, J. Geophys. Res. 106B, 515 (2001).
${ }^{94}$ A. Dewaele, G. Fiquet, D. Andrault, and D. Hausermann, J. Geophys. Res. 105B, 2869 (2000).
${ }^{95}$ P. Gillet, G. Fiquet, I. Daniel, and B. Reynard, Phys. Rev. B 60, 14660 (1999).
${ }^{96}$ P. I. Dorogokupets and A. R. Oganov (in preparation).
${ }^{97}$ R. A. Robie, B. S. Hemingway, and J. R. Fisher, U. S. Geol. Surv. Bull. 1452, 1 (1978).
${ }^{98}$ P. I. Dorogokupets and A. R. Oganov, Dokl. Earth Sci. 410, 1091 (2006).
${ }^{99}$ Y. Fei, J. Li, K. Hirose, W. Minarik, J. Van Orman, C. Sanloup, W. Van Westrenen, T. Komabayashi, and K. Funakoshi, Phys. Earth Planet. Inter. 143-144, 515 (2004).
${ }^{100}$ S. Merkel, H. R. Wenk, J. Shu, G. Shen, P. Gillet, H. K. Mao, and R. J. Hemley, J. Geophys. Res. 107B, 2271 (2002).
${ }^{101}$ Y. Akahama, H. Kawamura, and A. K. Singh, J. Appl. Phys. 92, 5892 (2002); 95, 4767 (2004).
${ }^{102}$ M. Matsui and N. Nishiyama, Geophys. Res. Lett. 29, 1368 (2002).
${ }^{103}$ N. Nishiyama, T. Irifune, T. Inoue, J. Ando, and K. Funakoshi, Phys. Earth Planet. Inter. 143, 185 (2004).
${ }^{104}$ Y. Fei, J. Van Orman, J. Li, W. Van Westrenen, C. Sanloup, W. Minarik, K. Hirose, T. Komabayashi, M. Walter, and K. Funakoshi, J. Geophys. Res. 109, B02305 (2004).
${ }^{105}$ T. Inoue, T. Irifune, Y. Higo, T. Sanehira, J. Ando, K. Funakoshi, and W. Utsumi, Phys. Chem. Miner. 33, 106 (2006).
${ }^{106}$ K. Hirose, R. Sinmyo, N. Sata, and Y. Ohishi, Geophys. Res. Lett. 33, L01310, doi:10.1029/2005GL024468 (2006).
${ }^{107}$ T. Tsuchiya, J. Geophys. Res. 108, 2462 (2003).
${ }^{108}$ O. L. Anderson, D. G. Isaak, and S. Yamamoto, J. Appl. Phys. 65, 1534 (1989).
${ }^{109}$ P. Souvatzis, A. Delin, and O. Eriksson, Phys. Rev. B 73, 054110 (2006).
${ }^{110}$ C. W. Greeff and M. J. Graf, Phys. Rev. B 69, 054107 (2004).
${ }^{111}$ A. R. Oganov and S. Ono, Nature (London) 430, 445 (2004); S. Ono, T. Kikegawa, and Y. Ohishi, Solid State Commun. 133, 55 (2005).
${ }^{112}$ Detailed tables with $x-T-P$ relations are available from the authors.
${ }^{113}$ N. Mounet and N. Marzari, Phys. Rev. B 71, 205214 (2005).

